Discussion of “A small investor model for the limit order book and some applications“ by Jörg Osterrieder

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A live order book

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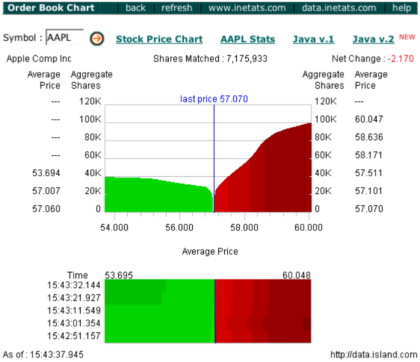
AAPL [Symbol Search]

Name Apple Comp Inc	**Market** NM
Last Price 57.3000	**Previous Close** 59.2400
Net Change -1.9400	**Percent Change** -3.27%
Shares Matched 6,603,448	**Shares Entered** 81,353,728
Orders Entered 402,668	**Open Orders** 397
Last Match Time 15:26:13.796	**Last Order Time** 15:26:14.663

BUY ORDERS		SELL ORDERS	
SHARES	PRICE	SHARES	PRICE
1,329	57.3000	300	57.3100
4,129	57.2900	3,680	57.3200
4,130	57.2800	2,600	57.3300
3,100	57.2700	3,571	57.3400
1,300	57.2600	2,291	57.3500
1,200	57.2500	950	57.3600
750	57.2400	1,100	57.3700
340	57.2200	400	57.3800
200	57.2000	300	57.3900
171	57.1800	249	57.4300
133 Buy Orders		264 Sell Orders	

A live order book II



Introduction

An order book entry consists of ... Type (buy/sell), limit price, size and time

This paper

- A general, dynamical model of the order book dynamics
- Derive statistical properties in different domains
- Applications:
  - Bid-ask spread
  - Execution probability

Why the order book?

- Because it contains a lot of information.
- Every order reveals an agent's preferences.
- In a perfect market, it is always optimal to reveal the true preferences.
- Limit orders are commitments → high credibility.

Measures for the order book

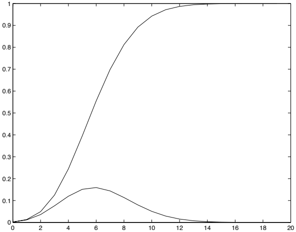
Measures of the order book are cumulative numbers of orders.

$$M_\theta\Big(\underbrace{[0,T]}_{\text{Time}}\times\underbrace{[0,p]}_{\text{rel. price}}\times\underbrace{[0,s]}_{\text{size}}\Big)$$

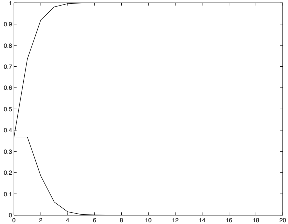
Choose a suitable joint density $f(T,R,S)$ for order arrival (and order cancellation).

Example (5.14): Orders arrive as a Poisson process at rate λ, size $s=1$, order survival probability is exponential decay: $P_{\text{surv}}(t)=\exp\left(-\frac{t}{\mu}\right)$.
$\rightarrow$ some complicated expression

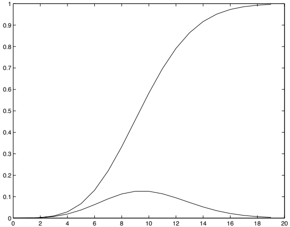
Example



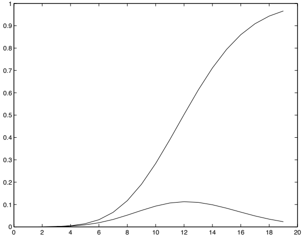
$P(k|\lambda = 10, \mu = 1, t = 1)$



$\mu = 0.1$



$t = 10$



$\lambda = 20$

Results

Bid-Ask spead

- Can be reproduced from model.
- Liquidity effect (as expected).

Heavy-tailedness of order book

- Heavy tailed arrival price distributions survive in the order book.
- Power law for distribution of relative prices shown.

Execution probability

market order	limit order
$P = 1$	$P < 1$
"bad" price	"better" price
utility u_m	utility u_l

- Optimal order submission schedule (type, size, limit, timing).

Quibbles

Very mathematical style.

- Simpler exposition possible? Not all symbols explained (and there are many!)
- No graphs.

Approximation for small investors. OK as a first step.

- Interesting to add large orders and impact on pice.
- (a) eat into the order book, (b) signaling effects.
- Applications:
  - Large funds (minimize impact)
  - Central banks (maximize impact)
  - Aim: order submission schedule

Do we really need the oder size s?

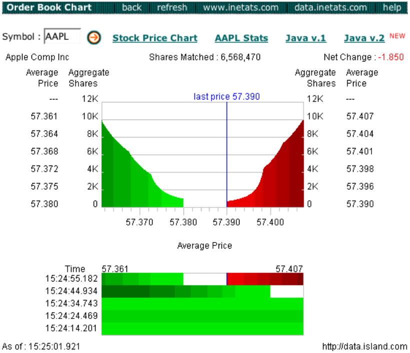
- Remember: $M_\theta\left([0,T] \times [0,p] \times [0,s]\right)$
- Small investors . . . s orders of size 1 just as good?

Quibbles 2

Order transmission delay and stochastic volatility.

- Volatility correctly reflected; increases order execution probability.
- What about prices change between limit decision and order placing?
- Large $\sigma^2 \rightarrow$ larger fraction gets executed immediately.
- This implies: $\lambda_{\text{agent}} = c \quad \Rightarrow \quad \lambda_{\text{exchange}} \propto \sigma^2$
- Changed (truncated) distribution?
- Small effect? (See Fig.)

Order-placing delay



order book for AAPL, $\Delta t = 49s$

Quibbles 3

Process for X_t explicitly modeled.

- The price process should be result of orders.
- Like to see:
  - endogenous X_t,
  - however not by explicitly modeling $f(\cdot)$,
  - but by starting from agents in a Lukas$^+$ economy.
- Possible results
  - flat or non-real order book
  - “Puzzle“
  - Add (behavioral?) model elements

Quibbles 4

Economic intuition, empirics and calibration.

- Results depend on model of order arrival process $f(\cdot)$
  - How much of the answer is in the assumptions?
  - Leave $f(\cdot)$ flexible $\rightarrow$ very general results
  - Make assumptions on $f(\cdot)$ $\rightarrow$ possibly unrealistic results
- What does order arrival mean economically?
  - Someone places an order, given the circumstances ξ
  - Need to use $f(\cdot, \xi)$, with ξ including volatility
- Suggestion:
  1. Use sufficiently flexible process for order arrival
  2. Calibrate using real world data
  3. Play around (e.g. check for parameter instabilities ...)