Discussion of
"Recovering Nonlinear Dynamics from Option Prices"
by A. Engulatov, R. Gonzalez and O. Scalliet

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The paper

Context

- Are affine latent variable models for option prices well specified?

In a nutshell

- Assume under $\mathbb{Q}$ a general price process for the underlying S_t

$$dS_t = rS_t + \sqrt{\sigma(v_t)}S_t dW_t + \text{jumps} \tag{1}$$

- Scalar process for volatility state variable

$$dv_t = \theta(v_t)dt + \sqrt{\eta(v_t)}dZ_t + \text{jumps} \tag{2}$$

with $\langle dW_t, dZ_t \rangle = \rho$

- Allow the following quantities to be general functions of v_t:
  - Mean reversion of volatility $\theta(v)$
  - Volatility of volatility $\eta(v)$
  - Jump intensity $\lambda(v)$
  - Note: $\sigma(v) = v_t$ for identification; ρ constant.

Results

Estimation:

- Approximate nonlinear functions via orthogonal base,
  e.g. $\theta^*(v) = \sum_{i=0}^{n} \theta_i P_i(v)$
- Estimation problem reduced to finding $\beta = \left(\theta_0 \dots \theta_n, \eta_0 \dots \eta_n, \lambda_0 \dots \lambda_n\right)$
- Pricing via Galerkin-Wavelet method
- Minimize squared pricing errors (use Tikhonov regularization)

Contribution:

- Framework for estimating a nonlinear volatility dynamics that nests known 1-factor models (Bates, Heston)
- Use of bounded basis functions (Chebyshev-polynomials)
- Estimation of nonlinear mean reversion, vol-of-vol and jump intensity
- Application of Galerkin-Wavelet method

Results

Main Results

- Some hints at misspecification of Bates (1996) model
- Saturation in $\theta(v)$, almost linear $\eta(v)$, flipping slope for $\lambda(v)$

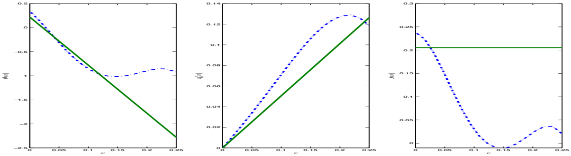


Figure 1: $\theta(v)$, $\eta(v)$, and $\lambda(v)$ for April – June 1997

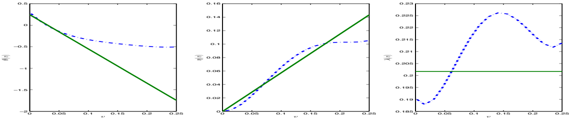


Figure 2: $\theta(v)$, $\eta(v)$, and $\lambda(v)$ for April – June 2002

Praise

- An interesting and new view on a long-debated topic
- Well-elaborated mathematics

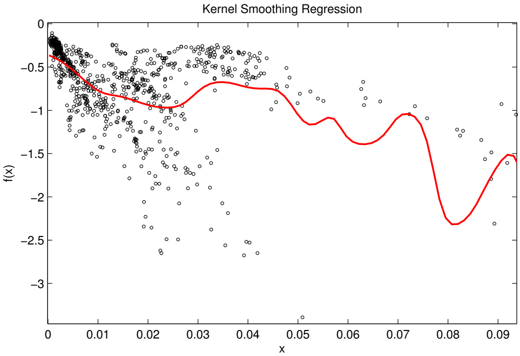
Comments (1)

Interpretation of results

- Core question: do results support non-linear or flexible specification?

Comparison to Bates (1996) model is not fair

- Bates (1996) has 8 parameters, here 18 (θ, η, λ) + 3 (jump size, ρ) $= 21$
- Multifactor affine models achieve flexible dynamics (with only 15 parameters under $\mathbb{Q}$) that exhibits saturation in vol-of-vol



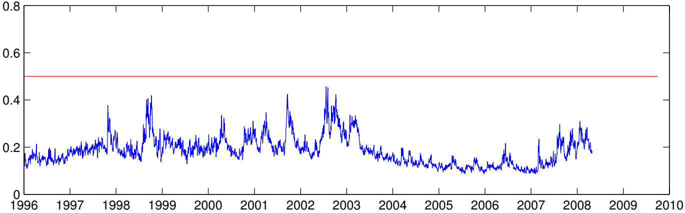
Comments (2)

Specification

- Is the large variability in $\lambda(\cdot)$ a sign that stochastic skewness is not well captured? Should ρ be time-varying?

Bounded support for v_t

- Central assumption of the paper
- Empirical exercise uses implied volatilities between 12% and 50%
- Is this a good assumption? Is this really necessary?



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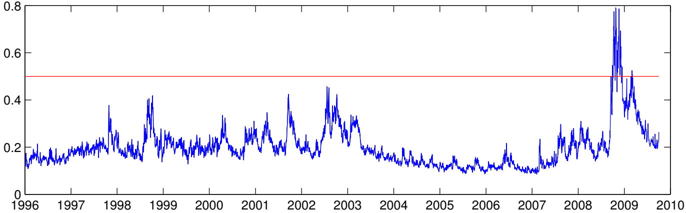

Comments (3)

Estimation

- Minimizing squared (dollar) pricing errors is not state of the art, consider using implied volatilities
- Standard errors could answer the interesting question: Are the parameters $\theta_i^*, \eta_i^*, \lambda_i^*$ significant for $i > 1$?
- Answer question of optimal n (possibly different for $\theta(v), \eta(v), \lambda(v)$)?
- Also interesting: confidence bounds around $\theta(v), \eta(v), \lambda(v)$
- This is definitely not easy ... no theory for distribution of errors.
- How is the upper end of the range ($v_t \to 0.25$) produced? If *total* variance is capped at 0.25, then *diffusive* variance will always be smaller.

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Time-varying parameters

- Parameters estimated quarter by quarter, differ sometimes heavily
- So is this $\theta_{\mathbf{t}}(v_t), \eta_{\mathbf{t}}(v_t), \lambda_{\mathbf{t}}(v_t)$?
  - YES. What are the dynamics of $\theta_t(\cdot), \eta_t(\cdot), \lambda_t(\cdot)$ and which risks are associated to the shifts in functional form?
  - NO. Will the nonlinearities persist, if one set of functions is estimated on the whole data set?

Comments (5)

Small quibbles

- Target audience? Style very mathematical. Some implementation details not explained.
- Symbol θ used twice (mean reversion, "other parameters" in estimation)
- Unclear price bid/ask interpolation: "As only bid/ask prices are available, we take the price consistent with the Black-Scholes volatility as actual option price".
- Report numerical results in tables

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My wishlist

- Economic analysis of the implications of the non-linear volatility dynamics in terms of implied volatility surface (level-skew-term structure) and pricing (dollar errors)
- Extended sample (longer period, more maturities, more strikes)
- Better econometrics with standard errors
- Dynamics under $\mathbb{P}$ with consistent estimation (particle filter?), some ideas of the market price of volatility/skewness risk, and hedging performance