Correlation risk and the term structure of interest rates

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The paper in a nutshell

- ☐ Reduced-form, continuous-time model of the term structure
- ☐ Affine setting, based on a new, matrix-valued (Wishart) process
- ☐ Closed-form solutions for short rate, yield curve, forwards, options
- ☐ Complex factor dynamics instead of complex market price of risk

Outstanding features:

- ☐ Tractable, flexible, yet parsimonious. Only 9 (18) parameters
- ☐ Match *first* and *second* moments of yields
- ☐ Stochastic second moments (with multiple factors driving them)
- ☐ Reproduce/explain a few regularities/ "puzzles"
  - – Switching sign of bond risk premium
  - – Predictability
  - – Cochrane-Piazzesi forecasting factor (3x3 model)
  - – Unspanned stochastic volatility

Matrix affine diffusion literature

Opening the field
Bru (1991), Wishart processes
Gourieroux, Sufana (2004) The Wishart Autoregressive Process of Multivariate Risk
Glickman, Philipov (2004, wp) Multivariate Stoch. Volatility Via Wishart Processes
daFonseca, Grasselli, Tebaldi (2007) A multifactor Heston model
Leippold, Trojani (2008, wp) Matrix Affine Jump Diffusion processes

Understand what is going on
$\rightarrow$ this paper
Buraschi, Porchia, Trojani (2008) Correlation Risk and Optimal Portfolio Choice
Gruber, Tebaldi, Trojani (wp) Option pricing with matrix affine jump diffusions

Discuss details
... yet to come ...

Wishart Process on $\mathcal{S}^+$ (positive definite, symmetric matrices)
$\longrightarrow$ perfect for modeling time-varying covariance matrices

Continuous-time:

$$d\Sigma_t = (kQ'Q + M\Sigma_t + \Sigma_t M')dt + \sqrt{\Sigma_t}dB_tQ + Q'dB_t'\sqrt{\Sigma_t}$$

Discrete time:

$$\begin{aligned}
\Sigma_{t+1} &= \sum_{i=1}^{k} X_{i,t+1}X_{i,t+1}' && (1)\\
X_{i,t+1} &= MX_{i,t} + Q\eta_{i,t+1} \qquad i = 1,..,k && (2)\\
\eta_{i,t+1} &\sim iiN(0, Id)\\
k &\geq dim(X)
\end{aligned}$$

Why the model works so well

Canonical $A_m(n)$ models

- State space $\mathbb{R}_+{}^m \times \mathbb{R}^{n-m}$
- Number of gaussian factors: tradeoff between forecasting and SV

This model

- State space $\mathcal{S}^n$
  - n diagonal elements; positive
  - $\frac{n(n-1)}{2}$ out-of diagonal elements; sign switching

Example 2×2*:* Apply PDP decomposition and rewrite slightly

$$\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & -\cos(\alpha) \end{pmatrix} \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ \sin(\alpha) & -\cos(\alpha) \end{pmatrix}^{-1}$$

Characterize Σ_t with $\lambda_1, \lambda_2, \alpha$ instead of $\Sigma_{11}, \Sigma_{12}, \Sigma_{22}$

- Rewrite state space as $\mathbb{R}_+^n \times [-1,1]^{\frac{1}{2}n(n-1)}$
  - n positive eigenvalues λ_i
  - $\frac{1}{2}n(n-1)$ angles α_i with cosines on $[-1,1]$

Note: could write process as vector-valued process, but rather messy.

Why the model works so well (2)

- All factors (λ_i, α_i) have stochastic volatility and are pairwise stochastically correlated.
- The stochastic volatility of all factors is multi-factor.
- The eigenvalues are conditionally independent (Bru 1990, (5.9)). Angles ensure multi-factor SV. Mean reversion of λ_i linked.

Elements of state load in

- **Prod. techn. volatility** – only eigenvalues: $\sqrt{Tr\Sigma_t} = \sqrt{\sum \lambda_i}$
- **Prod. techn. returns** – eigenvalues+angles: $Tr(D\Sigma_t) = \lambda_1(D_{11}\cos^2\alpha + D_{22}\sin^2\alpha) + \lambda_2(D_{11}\sin^2\alpha + D_{22}\cos^2\alpha) + \ldots$
- **Short rate** – eigenvalues+angles (similar)

Matrix form produces stochastic covariances of yields (21):

$$\frac{1}{dt} cov_t[dy_t^{\tau_1}, dy_t^{\tau_2}] = \frac{4}{\tau_1 \tau_2} Tr[A(\tau_1)\Sigma_t A(\tau_2) Q' Q]$$

Why the model works so well (2)

Extra note: Unspanned stochastic volatility

Yield of zero bond (20):

$$y_t^T = -\frac{1}{\tau}[b(\tau) + Tr(A(\tau)\Sigma_t)]$$

Interpretation: linear combination of elements of state.
$A(\tau)$ is part of the solution of the matrix Riccati equation.

Numeric result: 3×3 case, $\tau = 1$yr

λ_1	λ_2	λ_3
-0.12	-0.04	2.7E-6

Close-to singular structure of $A(\tau)$ suggests some factors do not load on yields $\rightarrow$ USV!

Possible improvements

Choice of k

Why $k = 3$, not $k = 7$ or open parameter? Why integer? (minor issue)

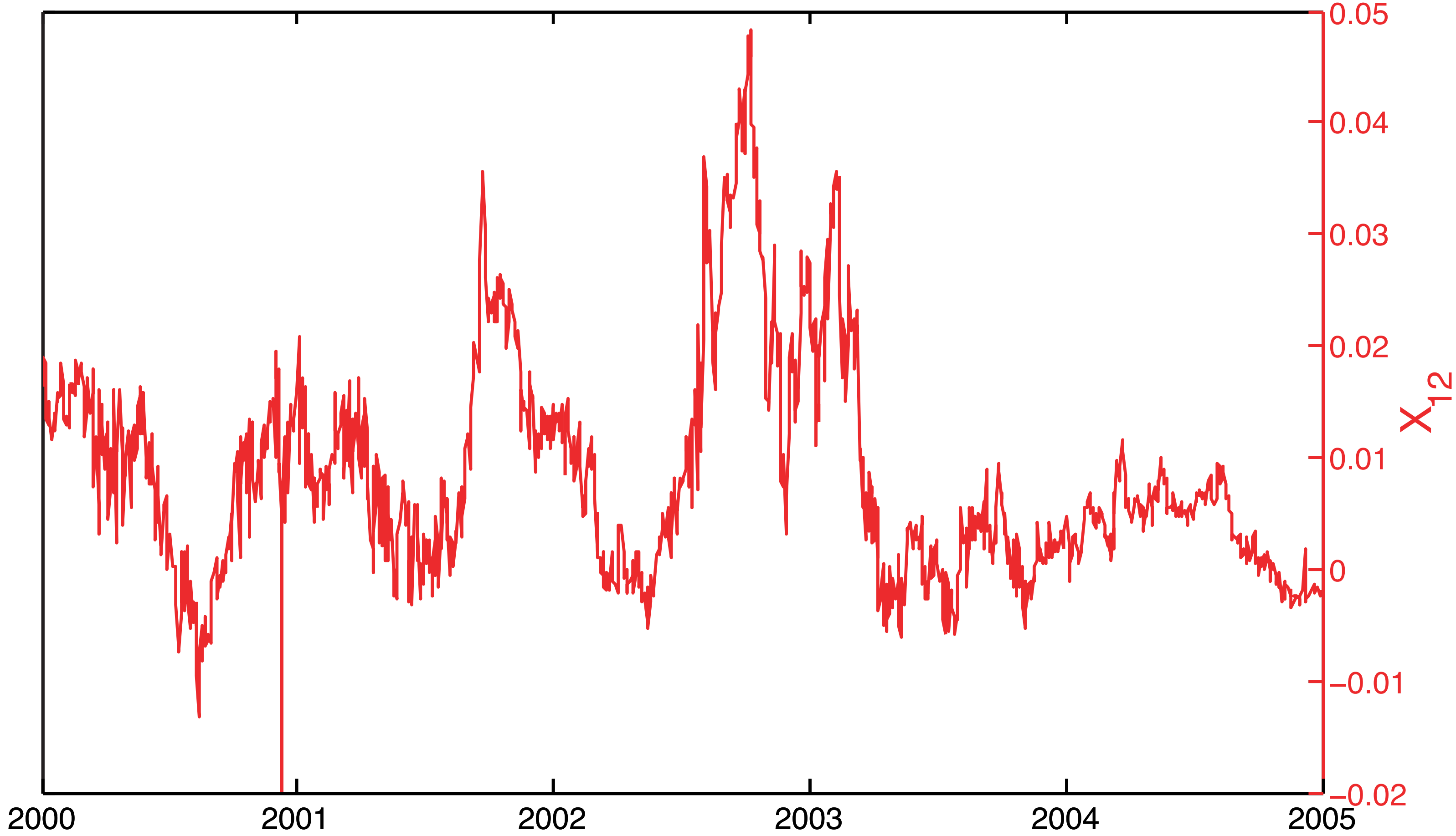
Illustrate the model

- Matrix (jump-) diffusion models are extremely rich, parametrization decides model behavior
- Use preferred parameters to identify leading terms $\rightarrow$ better understand the model

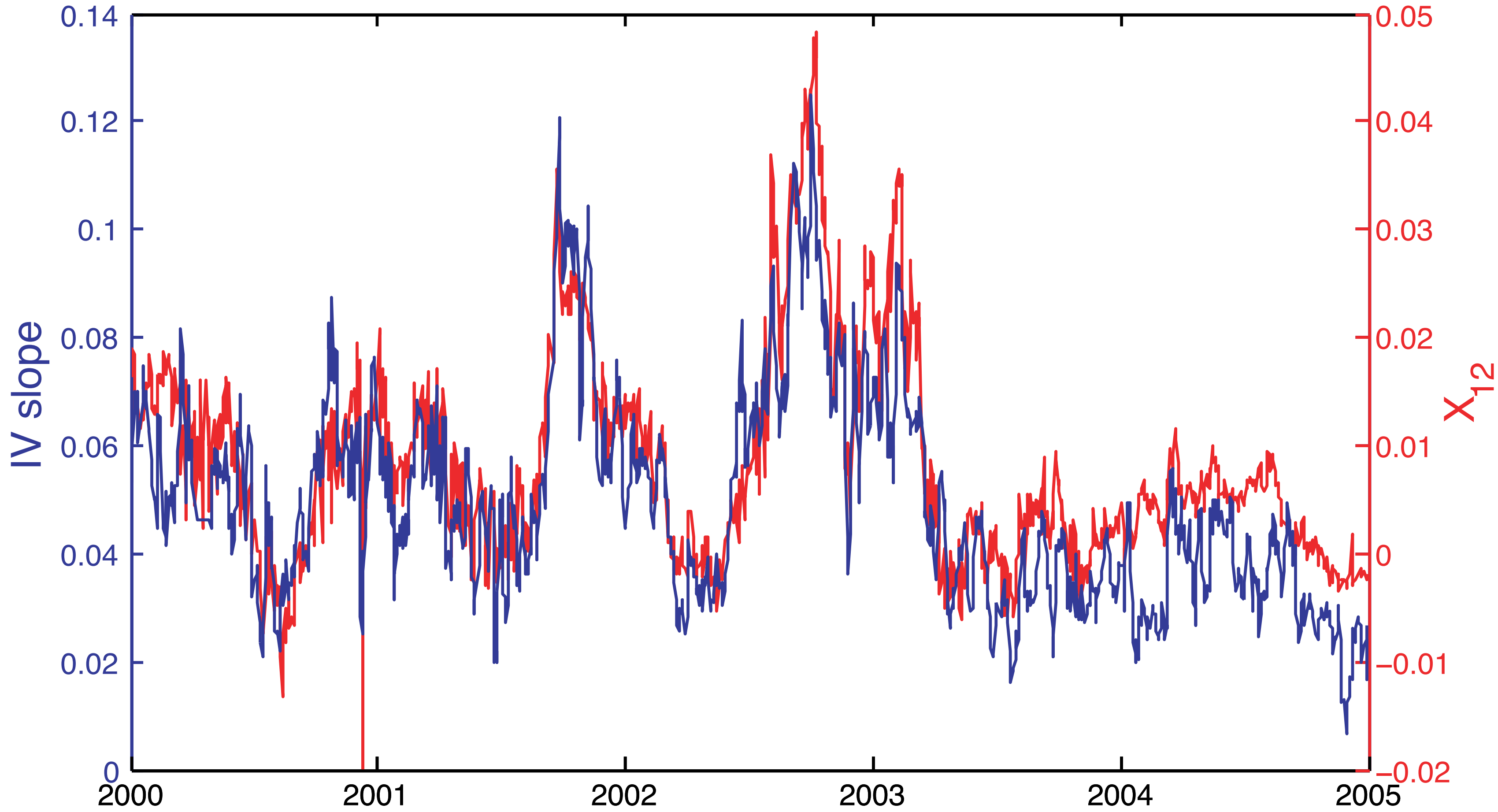
Show the state

- Already there: time series of 648 monthly states part of calibration
- Perform cross-sectional and time-series statistics on the state
- Facilitate approximations and interpretation
  Example: $\lambda_2 \ll \lambda_1 \rightarrow$ large range of out-of diagonal elements
- Reduced-form interpretation of state ($\rightarrow$ next slide)
- Economic interpretation: take Eq. 2 ("production technology") serious and close the loop between implied state and real economy

Example for reduced-form interpretation of state



Example for reduced-form interpretation of state (2)



Conclusion and appraisal