Political intergenerational risk sharing

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1. Introduction

In times of financial troubles, public PAYG pension systems come back into fashion. Large reductions in housing prices, losses in private pension funds, and increased volatility in the stock market have large negative effects on the private wealth of the elderly, and ultimately on their consumption. Real annuities—such as public pension benefits—may instead guarantee stable old age consumption, albeit typically at the cost of lower average returns on the contributions paid during the working period. In other words, PAYG pension systems entail an important intergenerational risk sharing component that proves crucial in periods of high financial instability.

This paper focuses on the role of the intergenerational risk sharing as a crucial motivation for the existence of social security systems. We characterize the optimal risk sharing mechanism introduced by a benevolent government and compare it to the social security system designed by office-seeking politicians, who choose the current risk sharing policy in order to win the elections — but cannot commit to future policies. We show that election-minded politicians typically prefer more spending in social security and introduce more persistent policies.

Since early contributions by Enders and Lapan (1982), Merton (1983), and Gordon and Varian (1988), PAYG social security systems have been recognized as an important instrument of intergenerational risk sharing. The demand for risk sharing stems from the uncertainty that is usually associated with endowments, wages and/or rate of returns on savings. Individuals would typically like to insure against bad realizations during their lifetimes, before they are even born. If there exists a long term player, such as a benevolent government that can bind current and future policies on the behalf of yet-to-be-born generations, intergenerational risk sharing through Social Security may arise.

A parallel, but less sympathetic literature provides instead evidence on the inefficiencies and the costs of the existing, generous social security systems. Large reductions in the employment rate among middle aged and elderly workers, rising labor cost, and the crowding out effect on the stock of capital induced by the reduction in private savings are only some of the by-products of these pension systems, which have been largely criticized. The upshot of this literature is that social security spending is inefficiently large.

Bohn (2003), and more recently Krueger and Kubler (2006), took a more comprehensive approach, and consider both these costs and benefits of PAYG schemes. They suggest that the crowding out effect on the private savings may be so severe as to overweight the positive role of intergenerational risk sharing. Storesletten et al. (1999) analyze the risk sharing properties of social security systems vis-à-vis idiosyncratic risks, such as wage fluctuations and mortality risk, and reach similar conclusions. Clearly, additional considerations on the labor market distortion induced by social security would only

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strengthen this argument. Yet, in a more general setting, which—
together with capital—features a long-lived asset (such as land),
Gottardi and Kubler (2006) provide more optimistic results on the
existence of a (ex-ante) Pareto improving social security system.
Hence, the jury is still out.

In this paper, we concentrate on the intergenerational risk sharing
property of a PAYG system vis-à-vis aggregate shocks, and abstract
from its many distortionary aspects. This approach may be relevant to
understand the historical experience of the US, where the introduc-
tion of the social security system followed the 1929 stock market
crash, and of several countries, such as Belgium, France, Germany, and
Italy, where episodes of hyperinflation wiped out the value of the
bonds issued in nominal terms and called for government's inter-
tervention to transform the existing (funded) systems into PAYG
schemes (see Flora, 1987). We show that both a benevolent
government and an office-seeking politician have an incentive to
introduce an unfunded system if a financial market crisis that wipes
out the private wealth of the elderly occurs.

We then turn to examining the main features of these risk sharing
instruments, and we ask whether electoral constraints may lead
politicians to choose “too much” social security spending. In our two-
period stochastic overlapping generation economy, the risk comes
from an aggregate shock to the stock market, which affects the net
private wealth, and thus the consumption of the agents in their old
age. A crucial feature of our setting is that individuals benefit from
intergenerational risk sharing, but redistributive motives are absent. A
benevolent government thus sets a risk sharing policy that spreads
current shocks forward into the future by trading off the well being
of current and future generations. The optimal linear risk sharing policy
features a transfer (typically) from the young to the old, which
depends negatively on the realization of the net private wealth of the
elderly.

Politicians' decisions are instead driven by electoral considera-
tions. We introduce a probabilistic voting environment in which
politicians choose the current social security policy by trading off the
well being of the currently alive generations of young and old
individuals. We concentrate on Markov perfect equilibria of this
probabilistic voting game, in which the equilibrium policy depends
only on the state of the economy. A specific feature of this political
equilibrium is that voting is dynamic: rational young voters anticipate
that current policies affect future policy decisions by inducing changes
in the state of the economy that shapes the incentives of the future
politicians and voters. As in the case of a benevolent government, this
mechanism allows for intergenerational risk sharing—as current
shocks are spread forward into the future—although in this case the
intergenerational tradeoff is driven by electoral considerations.

In our political setting, a PAYG system is more likely to be
introduced during an economic crisis, and to persist in future periods.
Its size depends crucially on the electoral strength—as measured by
the relative share of swing (or undecided) voters among the elderly—
of the old generation, who happens to face the crisis. In other words,
after a financial crisis office-seeking politicians are urged by their
electoral constraints to "bail out" the elderly through the provision of
generous public pensions. The politicians' incentives to intervene in
case of a negative shock effectively create a quasi asset—the PAYG
social security—whose returns are negatively correlated to stock
market returns. Interestingly, this policy turns out to be quite
persistent, since less disposable income for the current young
generation leads to lower net private wealth in their old age and
thus to more future government intervention.

We show that this political mechanism typically produces more
intergenerational risk sharing than the social optimum. Overspending
stems from the strategic behavior of the politicians under dynamic
voting. They exploit the expectations by current young voters, who anticipate that their current transfers will be compensated by
offsetting transfers provided by future politicians. This strategic effect

2 See Grossman and Helpman (1998), Hassler et al. (2003), but also Azzimonti (2005) and Battaglini and Coate (2008), among many others.
3 See also Song (2008) among others.
4 On this point, see also Casamatta et al. (2000) and Conde-Ruiz and Profeta (2007).

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comparative statics. Section 6 concludes. All the proofs are in the Appendix A.

2. A simple stochastic economy

We consider a two-period overlapping generations model of a small open economy. Every period two generations are alive: young and old. Population grows at a constant rate \( n \). Agents are endowed with one unit of labor in youth, which they supply inelastically to receive a wage, \( w \). Agents save their entire net endowment for old age consumption, which gives rise to a mean-variance representation:

\[
\text{output in the economy is given by}
\]

\[
y_t = wL_t + R_tK_t
\]

where \( K_t \) represents the stock of capital, i.e., the amount of savings, in the economy. Capital fully depreciates at every period. The return on capital is stochastic. Claims to capital represent the only (risky) asset and no serial correlation \( \rho \).

The distribution of the stochastic returns represents a crucial element in a model that analyzes intertemporal risk sharing. Instead of recurring to a specific distribution function, we choose to consider distributions that obey to two criteria. First, we assume that the average return from the risky asset is higher than the return from a hypothetical asset that promises to pay \( 1+\gamma \) in every future state of the world in exchange for a unit of resource today. Call \( \alpha \) the share of the youth unitary endowment invested in this asset. The optimization problem is:

\[
\max_{\{c_t\}} \frac{1}{2} E_t (c_{t+1} - \gamma)^2
\]

\[
c_{t+1} = R_{t+1}(1-\alpha) + \alpha (1+n)
\]

As the following first order condition shows, this amounts to a simple portfolio decision problem in which agents have to choose how to divide their savings between a safe asset that provides a return \( (1+n) \), and a risky asset with a stochastic return \( R_{t+1} =\alpha + (1-\alpha)R_{t+1} \):

\[
E_t (c_{t+1} - \gamma)(R_{t+1} - (1+n)) = 0
\]

The share of savings allocated to intergenerational risk sharing is

\[
\alpha^* = \frac{\gamma \sigma^2 + (R - (1+n))(R - \gamma)}{\alpha^2 + \gamma(1+n)}
\]

Depending on their degree of risk aversion, the young may have preferences for some intergenerational risk sharing. In particular, \( \alpha^* > 0 \) for a large enough degree of risk aversion, \( \gamma < \frac{-\gamma \sigma^2 + \gamma(1+n)}{\alpha^2 + \gamma(1+n)} \). where \( \alpha = \frac{\sigma^2 + R^2}{\gamma} \). To consider an environment in which intergenerational risk sharing plays a role, we thus set our next assumption.

5. In what follows, we will allow the benevolent government and the politicians to choose a negative transfer—that is, a transfer of resources from the elderly to the young—although for some realization of the shock. If this occurs, the savings of the young may exceed their labor income. However, it is straightforward to show that the consumption level implied by the equilibrium policy chosen by the benevolent government and the politicians (see Sections 3 and 4) never exceeds \( R + (1+n) \). The sufficient condition at Assumption 2 hence holds also for the following sections.

6. It is worth noticing that Assumption 3 is consistent with Assumption 2 for distribution functions that have a sufficiently high variance, that is, if \( \sigma^2 > (R - (1+n)) \).
Assumption 3. \(\gamma \frac{S-(1+n)R}{K-(1+n)}\).

It is immediate to see that under Assumption 3 any \(\alpha\) such that \(0<\alpha \leq \alpha^*\) implements an intergenerational risk sharing transfer scheme that is ex-ante Pareto improving.\(^7\)

Not surprisingly, in the presence of this intergenerational risk sharing transfer, the expected consumption in old age drops, \(E_t[c_{t+1}] = R - \alpha R - (1+n)\), but also the variance decreases, \(\text{Var}[c_{t+1}] = (1-\alpha^2)\sigma^2\). As a result, the coefficient of variation for the consumption is lower with this risk sharing device than in absence of risk sharing.

3. Intergenerational risk sharing by a benevolent government

In this section, we consider the intergenerational risk sharing decision of a benevolent government which cares about the well being of current and future generations. In every period, after the realization of the shock has occurred, and hence the net private wealth of the elderly has become known, a benevolent government decides whether to transfer resources from the young to the elderly (or vice versa).

The benevolent government optimization problem at time \(t\) is thus

\[\max_{\{T_t, \omega_t\}} U(c_t) + \delta(1+n)E_t U(c_{t+1}) + \delta^2(1+n)E_t U(c_{t+2}) + \ldots\]

subject to the budget constraint at Eq. (2.2), where \(\delta<1/(1+n)\) represents the benevolent government's discount rate, and the utility function is at Eq. (2.3). Individual agents take no economic decisions, while the government decision variable is the policy, \(T_t\). The state variable is \(\omega_t = R_1(1-T_{t-1})\), which characterizes the net private wealth of the elderly at time \(t\). We can thus use the following recursive formulation:

\[V(\omega_t) = \max_{\{T_t, \omega_t\}} \{U(\omega_t, T_t) + \delta(1+n)E_t V(\omega_{t+1})\}\].

(3.2)

The first order condition of this optimization problem with quadratic preferences as at Eq. (2.3) is:

\[-(\alpha_t + (1+n)T(\omega_t) - \gamma) + \delta E_{t+1}(\omega_t + (1+n)T(\omega_t + 1) - \gamma)R_{t+1} = 0.\]

(3.3)

The former term represents the marginal utility for the elderly of an increase in their consumption due to the intergenerational transfer, whereas the latter represents the expected reduction in marginal utility for the young from lower future consumption. To solve this optimization problem, we guess a linear policy, \(T(\omega_t) = A + B\omega_t\), and verify that it satisfies Eq. (3.3). Recall that \(S=\sigma^2 + R^2\). The next proposition characterizes the optimal interior linear policy function.

Proposition 3.1. If \(\delta \in A = \left\{\frac{1}{\delta}, \frac{1}{\delta^2} \right\}\), there exists a linear policy function \(T^{\omega} (\omega) = A + B\omega\) that solves the benevolent government problem at Eq. (3.1), with \(T^{\omega}(\omega) < 1 \forall \omega_t\) and \(T^{\omega}(\omega) > 0\) for some \(\omega_t\), where

\[B = -\frac{1}{\delta^2};\quad A = \frac{(\delta S - R\gamma) + (\gamma - (1+n)\cdot)}{\delta S - R(1+n)}\]

This proposition shows that if a benevolent government cares sufficiently about the future generations, i.e., if \(\delta \in A\), it will implement a linear interior intergenerational risk sharing mechanism, which provides the elderly with a transfer consisting of a constant share, \(A\). Which is reduced of a proportion \(B\) according to the realization of the state of the world, \(\omega_t\). In the worst case scenario, in which the elderly have zero private wealth, \(\omega_t = 0\) — for instance because of a very bad stock market shock, \(R_t = R = 0\) — the benevolent government imposes a positive, large transfer on the young, \(T^{\omega}(\omega_t) = A = (0,1)\). Better realizations of the rate of return, and hence higher private wealth for the elderly, are associated with lower transfers from the young.

A brief discussion of the restrictions on the benevolent government's discount factor is in order. A low weight on the future generations, \(\delta < 1/R\), may lead, in the occurrence of a particularly negative shock on the returns, to a complete transfer of resources from the young to the elderly, \(T^{\omega}(\omega_t) = 1\). Equilibria with full expropriation have the unpleasant feature of representing an absorbing state. Indeed, the young generation on which a 100% tax rate is imposed reaches old age with zero private wealth, \(\omega_t = 0\), which will in turn command a 100% tax rate on the young and so on. In the remaining of the paper, we will disregard these full expropriation equilibria and concentrate on interior equilibrium solution, thereby assuming that \(\delta > 1/R\). If the weight on the future generations is too large, however, this would call for a transfer from the old to the young even in the worst case scenario, in which \(\omega_t = 0\), which is clearly unfeasible. As shown in the Appendix A, this case is ruled out for \(\delta < 1/(1+n)\).

This former restriction on \(\delta\) plays another important role: it allows us to abstract from the redistributive motive that may lead the benevolent government to set a transfer from the young to the old. In fact, as shown in the following Lemma, in the deterministic version of our model economy (i.e., for \(\sigma = 0\), \(R_t = R\)), in which no intergenerational risk sharing motive can be in place, a benevolent government that cares sufficiently about the young, that is, if \(\delta > 1/R\), would not transfer resources to the elderly.

Lemma 3.2. In the deterministic environment associated to our model economy, in which \(R_t = R\) for any \(t\) and \(\sigma^2 = 0\), it holds \(T^{\omega} \leq 0\) whenever \(\delta \geq 1/R\).

The next proposition further characterizes this interior equilibrium risk sharing policy by presenting the results of some comparative statics.

Proposition 3.3. For \(\delta \in A\) an increase in (i) the discount factor, \(\delta\); or in (ii) average rate of return, \(R\), reduces the fixed component, \(A\), of the linear policy function \(T^{\omega}(\omega_t)\), and makes the transfer less responsive to the shock. An increase in the variance of the shock, \(\sigma^2\), increases the linear policy function \(T^{\omega}(\omega_t)\).

The intuition is straightforward. Recall that for any given realization of the shock, providing a transfer to the current elderly comes at the cost of lower expected utility for the young generations, because of the opportunity cost of using a PAYG system for risk sharing — given that \(R = 1 + n\). Hence, the higher the weight placed on these future generations, or the higher the average return of the risky asset—and hence the opportunity cost—the lower the fixed component of the transfer, which becomes also flatter. On the other hand, higher
volatility of the returns clearly requires more risk sharing, and hence $T^C(\omega_t)$ increases.

4. Intergenerational risk sharing by office-seeking politicians

In the political system, intergenerational risk sharing may arise because office-seeking politicians choose to transfer resources from the young to the old (or vice versa) in order to improve their electoral perspective. Politicians act after the stock market shock has occurred—and hence the return on the risky asset, $R_t$, is realized.

Formally, we consider a probabilistic voting model, (see Persson and Tabellini, 2000; Hassler et al., 2003, for a framework with dynamic voting). Two political candidates compete in a majoritarian election. Each candidate determines her political platform, which is represented by the contribution, $T$, in order to maximize her probability of winning the election. The candidate who wins the election becomes the policy-maker, and implements the proposed policy. Elections take place every period, after the realization of the stochastic return on the assets of the current old. Hence, political candidates can condition their intergenerational risk sharing policy on the realized state of the world.

At every election, individual’s voting decisions depend on the policy chosen by the political candidates—and thus on how this affects their utility, on the individual’s political ideology towards the two candidates, and on a common popularity shock that may hit the candidates before the election. Political candidates will use the intergenerational risk sharing policy to target the young and/or the old, in an attempt to increase their probability of winning the election, but they cannot affect the voters’ ideology or their own popularity. Within each age group, all individuals share the same economic preferences, thereby being equally affected by the candidates’ platforms. Elderly care only about the current transfer. Instead, the preferences of the current young—and thus their voting behavior—depend also on the expected future policy. If the young were myopic, they would only consider the direct effect of the current payroll tax on their endowment and thereby on their future consumption. Young workers however do realize that a current tax makes them more likely to be poorer tomorrow, and this may modify the future politicians’ behavior. Current young electors hence need to understand and evaluate how the decisions of the current politicians may affect the future politicians’ policy choice. We choose to consider a Markov policy, in which intergenerational risk sharing transfers depend only on the current state of the economy, in order to emphasize the absence of commitment to future policies by the politicians.

Besides the utility provided by the economic policy, individuals care also about the political ideology, with some people feeling ideologically closer to one candidate or another. The distribution of ideology within each age group affects the candidate policy decision by determining the size of the swing voters, i.e., of the non-ideological voters who can be convinced to vote for a candidate if targeted with the appropriate policy. It is convenient to assume that each age group has a uniform distribution of ideology across agents.

In this environment, the two political candidates face the same optimization problem, and thus their political platforms converge, i.e. both candidates choose the same contribution, $T$. Maximizing the probability of winning the election at time $t$ is equivalent to maximizing the following expression, which may also be interpreted as the welfare function of the policy-maker at time $t$:

$$W_t = \phi_y U(c_t) + (1 + n)\phi_y E_U(c_{t+1})$$

(4.1)

where $\phi_y$ and $\phi_o$ represent the density of the uniform ideology distribution function in the two groups, respectively old and young. We normalize $\phi_y = 1$ and define $\phi = \phi_y = 0$ as the relative importance of non-ideological, or swing, voters among the young generation. This can be interpreted as a measure of how fiercely the young generations pursue their interests in the political arena.

Eq. (4.1) shows that political competition, as modelled in this probabilistic voting framework, entails a tradeoff between providing state contingent transfers (and utility) to current retirees and providing current negative transfers, but expected positive transfers (and utility) to current workers. Hence, the voting behavior of the young depends on the policy chosen by the current politician, as well as on its impact on tomorrow’s policy. To model this intertemporal link, we focus on stationary Markov perfect equilibria, in which each politician’s policy decision is contingent on the current state of the economy. At any time $t$, the state variable is the amount of old age consumption that can be financed out of the private assets $\omega_t$. This clearly depends on the young’s savings (or net income) and on the outcome of the stock market. Thus, past policies directly contribute to defining the state of the economy. Clearly, each politician anticipates that its current choice will affect the incentive faced by the future politicians and, therefore, the future level of the intergenerational risk sharing.

The optimization problem of a policy-maker at time $t$ is thus

$$\max_{\{\omega_t \leq T(\omega_t) \leq 1\}} U(c_t) + \phi(1 + n)E_U(c_{t+1})$$

(4.2)

where the Markov strategy is $T_t = T(\omega_t)$, the state variable is defined as $\omega_t = R_t(1 - T_{t-1})$. Consumption can be written as $c_t = \omega_t + (1 + n)T_{t-1}$. Notice that $c_{t+1} = R_{t+1}(1 - T_{t+1}) + (1 + n)T_{t+1}$, where $T_{t+1}$ is the expected strategy played by future governments.

We can now formally define the linear Markov policy analyzed in this section.

**Definition 4.1**: A policy $T^*_t(\omega_t) = \theta + T^*_t(\omega_t)$, where $\theta$ and $T^*_t(\omega_t)$ are constant parameters, is a linear Markov perfect equilibrium of the intergenerational risk sharing game if it is a fixed point of the mapping from $T^*_t(\cdot)$ to $T^*_t(\cdot)$, where $T^*_t(\cdot)$ is the expected policy function,

$$T^*_t(\omega_t) = \arg \max_{T(\omega_t)} U(\omega_t + (1 + n)T(\omega_t)) + \phi(1 + n)E_U((1 - T(\omega_t))R_{t+1} + (1 + n)T(\omega_{t+1}))$$

and $T^*_t(\omega_t) = T(\omega_t)$.

In what follows, we will characterize this equilibrium policy outcome for any well behaved utility function with $U^\prime > 0$ and $U^\prime \prime > 0$. We will return to the quadratic utility function later in this section.

The first order condition for the politician’s problem is

$$U(c_t) + \phi E_U(c_{t+1}) = 0$$

(4.3)

where—for $T^\frac{\prime \prime}{\prime} > 0$—the former term represents the marginal utility of increasing the consumption of the current old, while the latter defines the expected marginal disutility to the current young from imposing this risk sharing policy. This marginal cost can be decomposed as follows:

$$\frac{\partial E_U(c_{t+1})}{\partial T^*_t(\omega_t)} = E_t U(c_{t+1}) \left[1 + (1 + n)\frac{\partial T^*_t(\omega_t)}{\partial \omega_t} + 1\frac{\partial \omega_t + 1}{\partial T^*_t(\omega_t)}\right] \frac{\partial \omega_t + 1}{\partial T^*_t(\omega_t)}$$

Notice that the impact of today’s policy on tomorrow net private wealth is $\frac{\partial \omega_t + 1}{\partial T^*_t(\omega_t)} = -R_{t+1}$ and defines $\frac{\partial T^*_t(\omega_t)}{\partial \omega_t} = T$. The first order condition of the maximization problem at Eq. (4.2) becomes:

$$U(c_t) - \phi \left[1 + (1 + n)T^\prime\right] E_t U(c_{t+1}) R_{t+1} = 0$$

(4.4)

Thus, if an interior (linear) Markov equilibrium policy $T^*_t(\omega_t)$ exists, it must satisfy $-1/(1 + n) < T(\omega_t) \leq 0$. The above expression

\[ T^\prime(\omega_t) \leq 0, \text{ consider the impact of a small increase in } \omega_t \text{ on } E_t U(c_{t+1}) \text{ for } T(\omega_t) > 0. \text{ If } T(\omega_t) \text{ were positive, } c_t \text{ would increase and } c_{t+1} \text{ decrease, so that Eq. (4.4) would no longer hold with equality.} \]
provides a first insight on this political intergenerational risk sharing. This policy is shaped by the political tradeoff between bailing out the current old from a negative stock market shock and imposing an expected cost on current young. In an interior Markov equilibrium, the intergenerational risk sharing agreement\textsuperscript{10} features a transfer from the young to the old that is inversely related to the outcome of the stock market. It is important to notice that the political discretion by policy-makers in setting an intergenerational transfer policy creates a quasi asset, whose returns are negatively correlated to stock market returns. Furthermore, by increasing \( T^0 \), the current politician reduces, for any future realization of the stock market \( R_t + 1 \), the level of private wealth of the current young, thereby requiring larger future intervention. This property creates a strategic effect that induces persistence in the policy. In this model, a large current political intervention creates its own constituency for future large political interventions. Although this is commonly thought as the root of the persistence of inefficient policies (see Coate and Morris, 1999; Conde-Ruiz and Galasso, 2003), in our context the tension between persistence and efficient allocation of risk is more subtle. The essence of intergenerational risk sharing is to spread current shocks on to future generations (see also Gordon and Varian, 1988; Ball and Mankiw, 2007), i.e., persistence is a crucial ingredient of an efficient risk sharing policy. By transferring the burden of current negative shock to current workers, the politician triggers a reaction by all future politicians, who keep transferring this shock into the infinite future.

To obtain further insights on the intergenerational risk sharing policy chosen by office-seeking politicians, we continue our analysis using the quadratic utility function described at Eq. (2.3). The first order condition of the maximization problem at Eq. (4.2), which describes the stationary Markov policy chosen by the politician a time \( t \), becomes:

\[
-\left[ \alpha_1 + (1 + n)T^0(\omega_t) - \gamma \right] + \phi \left[ 1 + (1 + n)T^0 \right] E_t \times \omega_{t+1} + 1 + (1 + n)T^0(\omega_{t+1}) - \gamma \right] R_{t+1} = 0.
\]

subject to the linear policy \( T^0(\omega) = \theta + \omega/\mathcal{R} \). The next proposition characterizes the equilibrium linear policy function.

**Proposition 4.2.** If \( \phi(\omega; \mathcal{R}) = \left( \frac{S}{\mathcal{R} + n + \mathcal{R}(1 + n)} \right)^{\gamma} \left( \frac{S - \mathcal{R}}{\mathcal{R} + n + \mathcal{R}(1 + n)} \right)^{1 - \gamma} \), there exists a linear Markov perfect policy function \( T^0(\omega) = \theta + \omega/\mathcal{R} \), with \( T^0(\omega) < 1 \) for \( \omega \), and \( T^0(\omega) > 0 \) for some \( \omega \), where

\[
\theta = \frac{\phi(S - \mathcal{R} - (1 + n))}{\phi(S - \mathcal{R} + 1 + n)) + \left( 1 + 4(1 + n) \right) \phi \mathcal{S}}
\]

This proposition characterizes the behavior of the sequence of office-seeking politicians under a Markov perfect equilibrium when individual preferences have a mean-variance representation. Analogously to the case of a benevolent government, the conditions on \( \phi \)

\begin{itemize}
  \item make sure that the young generation has sufficient relative electoral weight to avoid equilibrium sequences featuring full expropriation of the young, in the occurrence of negative stock market shocks. Also in this case, full expropriation would become an absorbing state, since it would lead the young to have zero private wealth in old age, \( \omega = 0 \), and thus trigger further full expropriation by future old generations.
  \item For a higher electoral weight of the young, office-seeking politicians will still introduce a linear intergenerational risk sharing scheme, featuring a positive constant component, \( \theta \), which is reduced by a share \( \mathcal{T} \) according to the realization of the state of the world, \( \omega \). In the worst case scenario, \( \omega = 0 \), the elderly obtain the largest transfer \( T^0(\omega) = \theta \in (0, 1) \). Higher levels of private wealth, \( \omega \), command lower transfers. Finally, if the relative electoral weight of the young is large enough, that is, if \( \phi \) is above the upper limit of \( \Phi \), politicians would refrain from introducing a (PAYG) intergenerational risk sharing system, even when the worst case, \( \omega = 0 \), occurs.
  \item As with the benevolent government, the restrictions on \( \phi \) and in particular the fact that the political weight of the young is above the lower threshold of \( \phi \) guarantee that no redistributive motive shapes the incentives of the politicians when setting the transfer policy from young to old. The following lemma in fact establishes that in the deterministic version of our economy, no transfer from the young to the old would take place if \( \phi = \phi^* \).
\end{itemize}

**Lemma 4.3.** In the deterministic environment associated to our model economy, with \( \mathcal{R} = \mathcal{R} \) and \( \phi^2 = 0 \) for any \( t \), \( T^0 \leq 0 \) whenever \( \phi = \phi^* \).

**Proposition 4.4.** For \( \phi = \phi^* \), an increase in the average rate of return, \( \mathcal{R} \), or in political weight of the young, \( \phi \), modifies the linear policy function \( T^0(\omega) = \theta + \omega/\mathcal{R} \) by decreasing its fixed component \( \theta \) and by making it less responsive to the realization of the state variable, \( \omega \).

The intuition is straightforward. A lower average return reduces the (opportunity) cost--recall that \( \mathcal{R} > 1 + n \)--of using a PAYG system to provide risk sharing, while an increase in the political weight of the young, \( \phi \), increases the electoral cost of introducing risk sharing. In both cases, the fixed component of the system thus shrinks, and the system becomes less responsive to the shocks. In other words, the young prefer less risk sharing with a flatter schedule.

5. How well do politicians do?

Both office-seeking politicians and a benevolent government would provide intergenerational risk sharing in the stochastic environment\textsuperscript{11} introduced at Section 2. Moreover, the linear equilibrium policies share similar properties in the two cases. As characterized at Propositions 3.1 and 4.2, both policies consist of a constant component (A for a benevolent government and \( \theta \) for the politicians), which is transferred to the elderly in the worst case scenario, i.e., for \( \omega = 0 \), and of a proportion–respectively \( B \) and \( T \)–which reduces the maximum transfer in accordance to the realization of the state variable \( \omega \). Propositions 3.3 and 4.4 push these similarities even further, as they suggest that the steady state properties of the two policy functions are comparable.

We now examine under which conditions politicians aiming at being elected behave exactly as a benevolent government. In other words, when is the interior linear Markov equilibrium policy chosen by office-seeking politicians optimal? The next proposition characterizes when the interior linear Markov equilibrium policy \( T^0(\omega) \) coincides with the optimal policy, \( T^0(\omega) \). A graphic representation is at Fig. 1.

\textsuperscript{10} The structure of the problem faced by these office-seeking politicians shares some structural features with the problem of optimal bequests strategies in altruistic economies where the current generation cares about the utilities of their immediate successors (see Phelps and Pollak, 1968; Bernheim and Rey, 1989, and, more recently, Nowak, 2006, and references therein). The main difference is that in our political environment the weight on different generations depends on the relative share of non-ideological (swing) voters in each age group; whereas in the former class of models the relative weight between (state contingent) utility to ancestors and expected utility to descendants is dictated by altruism and other ethical considerations.

\textsuperscript{11} Neither one would however transfer resources from the young to the old for redistributive motives in the deterministic version of our economic environment.
once they are born, and uncertainty is realized, there is no more room before they are even born. Clearly, this is not contractible upon. Yet, individuals would bene

Proposition 5.1. For \( \delta \in \Lambda \) and \( \phi \equiv \phi \), if \( \phi = f(\tilde{\delta}) = \delta[1 - \frac{1}{\omega}]^{-1} \), the interior linear Markov equilibrium policy chosen by office-seeking politicians correspond to the optimal policy, i.e., \( T^G(\omega) = T^G(\omega) \forall \omega \). For \( \phi = f(\delta) \), \( T^G(\omega) > T^G(\omega) \forall \omega \), with \( T^G > B \) and \( \theta > A \). For \( \phi = f(\delta) \), \( T^G(\omega) < T^G(\omega) \forall \omega \), with \( T^G < B \) and \( \theta < A \).

According to this proposition, a Markov game among successive generations of office-seeking politicians may deliver the optimal policy only if the relative electoral weight of the young is larger than the weight assigned by a benevolent government to the future generations, since \( \phi = f(\tilde{\delta}) > \delta \). For equal weights, \( \phi = \delta \), office-seeking politicians will provide larger transfer than the social optimum. This transfer is characterized by a larger than optimal fixed component, \( \theta > A \); and by a lower than optimal reduction associated to the state of nature, \( T^G > B \). In other words, political intergenerational risk sharing policy is too generous, and too persistent, that is, not enough responsive to the state variable. These distortions come from the politicians’ strategic behavior. In their decisions over the transfer policy, current politicians anticipate that future politicians will compensate the current young in their old age for their current social security benefits, this stems from the fact that higher taxes on today’s environment lead to a lower private wealth in old age—that is, to a lower state variable in the following period—thereby requiring more transfers by the future politicians. The policy response of the future politicians thus reduces the current (electoral) cost of transferring resources to the elderly, and leads to overspending — unless the young enjoy an unusually large political power, i.e., \( \phi > f(\delta) \).

These two intergenerational risk sharing policies have different implications for the consumption in old age. In both cases, old age consumption depends on the realization of the shock to the returns of the risky assets. However, for \( \phi = f(\tilde{\delta}) \), that is, when the politicians are more generous than optimal in their risk sharing policy, they guarantee a higher than optimal expected consumption in old age, but at the cost of introducing also a higher than optimal variance of consumption. By transferring too many resources to old age, and by failing to have these transfers depending more on the realization of the state variable, the politicians fail to provide the optimal risk sharing policy.

6. Concluding remarks

The risk sharing properties of social security have long been recognized in the literature. In several stochastic environments, individuals would benefit from insuring against aggregate shocks before they are even born. Clearly, this is not contractible upon. Yet, once they are born, and uncertainty is realized, there is no more room for risk sharing. Establishing a PAYG system thus seems to require the existence of a long term player, who can bind future, yet-to-be-born generations to carry out the risk sharing policy.

We show that both a benevolent government, who can bind future generations, and office-seeking politicians, who cannot, choose to adopt a state contingent social security system with analogous features. The amount of resources transferred to the elderly by the working generation depends negatively on the elderly private wealth — and therefore on the realization of the aggregate shock to the returns of the risk asset. This state contingent social security thus constitutes a quasi asset, whose returns are negatively related to the market returns. This result is in line with Ball and Mankiw (2007) who propose an optimal intergenerational risk sharing plan featuring a negative correlation between social security benefits and asset returns.

Despite these similarities, the intergenerational risk sharing schemes proposed by a benevolent government and by the politicians may also differ. Office-seeking politicians are more likely to provide generous transfers that are less responsive to the aggregate shock, and hence more persistent. While persistence is typically at the root of efficient intergenerational risk sharing policies, since it allows to spread the risk over time and hence over several generations, office-seeking politicians have an incentive to overplay this feature. In fact, politicians are willing to tax more heavily current workers and to provide generous transfers to the current elderly, because they anticipate that future politicians—facing elderly individuals with low net wealth, due to the large contributions they had to pay in their youth—will compensate them with generous pension transfers. This mechanism thus reduces the electoral cost among the young voters of providing large transfers and leads to generous, persistent pension systems.12

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Appendix A

Proof of Proposition 3.1. Consider the optimization problem of the benevolent government at Eq. (3.1). Its recursive formulation yields the following first order condition:

\[
\frac{\partial U'(G_t)}{\partial I_t} + \delta \frac{\partial U'(G_{t+1})}{\partial \omega_{t+1}} \frac{\partial \omega_{t+1}}{\partial I_t} = 0
\]

where \( \omega_n = R_t(1 - T_n - 1) \) defines the state variable at time \( t \). Using the utility function at Eq. (2.3), the above expression can be written as Eq. (3.3). To solve for interior equilibrium policies, we guess a linear solution: \( T^G(\omega_n) = A_n + B_\omega \). Using simple algebra, from Eq. (3.3) we obtain the following expression:

\[
T^G(\omega_n) = \frac{-\omega_n}{1 + n + \delta S(1 + B(1 + n))} + \frac{\gamma (1 - \delta R) + \delta S(1 + B(1 + n)) - \omega_n (1 + n)/\delta R}{1 + n + \delta S(1 + B(1 + n))}
\]

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12 This result for example is consistent with Bohn (2003) findings that the current US social security system does not provide the optimal level of risk sharing, since it is too generous with the elderly and shifts most of the burden of risk sharing on to future generations.
Hence, we have

\[ B = \frac{1}{1 + n + \delta S(1 + B(1 + n))} \quad (6.1) \]

\[ A = \frac{\gamma(1 - \delta R) + \delta S(1 + B(1 + n)) - \delta(1 + n)AR}{1 + n + \delta S(1 + B(1 + n))} \quad (6.2) \]

For \( 1 + B(1 + n) \neq 0 \), we obtain

\[ B = \frac{1}{\delta S} \quad (6.3) \]

\[ A = \frac{\gamma - (1 + n) + \delta(S - \gamma R)}{\delta S - (1 + n)R} \quad (6.4) \]

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, that is, \( T^0(\omega) < 1 \forall \omega \). To guarantee that this condition holds, we need to impose \( T^2(\omega) < 1 \) for \( \omega = 0 \), i.e., \( A < 1 \). Simple algebra yields \( \delta > \sqrt{R} \).

Additionally, we require some risk sharing to take place, and thus a transfer from the young to the old to occur at least in some state. The most favorable case for this transfer to occur is for \( \omega = 0 \). Hence we need to have \( A > 0 \). Simple algebra shows that, since \( \gamma > \sqrt{R} \) (Assumption 2), a sufficient condition for \( A > 0 \) is \( \delta > \frac{\sqrt{R} - 1}{\sqrt{R} + 1} \). However, using Assumption 3, it is easy to see that \( \delta < \frac{\sqrt{R} - 1}{\sqrt{R} + 1} \) implies \( \delta < \frac{\sqrt{R} - 1}{\sqrt{R} + 1} \), and thus \( A > 0 \). Q.E.D.

**Proof of Lemma 3.2.** A stationary Markov policy \( T \) for \( R_0 = R \) and \( \sigma^2 = 0 \) satisfies \( T = A + B(1 - T)R \). Downward transfers from old to young occur if \( A + BR \leq 0 \). Substituting the expressions for \( A \) and \( B \) from Proposition 3.1, we have that \( \delta \geq \sqrt{R} \).

**Proof of Proposition 3.3.** For \( \delta \in \Lambda \) recall that the linear policy function is \( T^0(\omega)_t = A + B_0 \) with \( A \) and \( B \) defined in Proposition 3.1.

(i) Consider a change in the discount factor, \( \delta \). Simple algebra shows that \( \frac{\partial A}{\partial \delta} = \frac{1}{\delta S} > 0 \) and \( \frac{\partial B}{\partial \delta} = -\frac{\gamma(1 + n) + \delta S(1 + B(1 + n)) - \delta(1 + n)AR}{1 + n + \delta S(1 + B(1 + n))} < 0 \).

(ii) Consider a change in the average rate of return, \( R \). We have that \( \frac{\partial B}{\partial R} = \frac{1}{\delta SR} > 0 \) and \( \frac{\partial A}{\partial R} = -\frac{(\gamma - (1 + n) + \delta(S - \gamma R))}{\delta S - (1 + n)R} < 0 \), since \( \sigma^2 > R^2 \) (Assumption 1).

Finally, (iii) consider a change in the variance of the shock, \( \sigma^2 \). It is easy to see that \( \frac{\partial B}{\partial \sigma^2} = \frac{1}{\delta S} > 0 \) and \( \frac{\partial A}{\partial \sigma^2} = \frac{(\delta S - (1 + n)R) - \delta(1 + n)(\sigma^2 - (1 + n)R)^2}{\delta S - (1 + n)R^2} > 0 \). Hence, \( \frac{\partial T^0(\omega)_{\delta}}{\partial \delta} > 0 \). Q.E.D.

**Proof of Proposition 4.2.** Consider the first order condition at Eq. (4.5), which describes the stationary Markov policy chosen by the politician a time. Recall that the state variable is defined as \( \omega_t = R_t(1 - T_{t-1}) \) \( \forall t \), and that \( T^0(\omega) = \theta + T \omega \). Moreover, define \( Q_t = 1 + (1 + n)T \). We need to obtain the values of the parameters \( T^0 \) and \( \theta \), which solve this FOC. Using simple algebra, from Eq. (4.5) we obtain the following expression:

\[ T^0(\omega) = \frac{1}{1 + n + \delta S Q^2} + \frac{\gamma + \delta S Q^2 - \delta Q R(\gamma - \theta)(1 + n))}{1 + n + \delta S Q^2} \]

Hence, we have

\[ T = \frac{1}{1 + n + \delta S Q^2} \quad (6.5) \]

\[ \theta = \frac{\gamma + \delta S Q^2 - \delta Q R(\gamma - \theta)(1 + n))}{1 + n + \delta S Q^2} \quad (6.6) \]

Since \( Q = 1 + (1 + n)T \), we solve the expression at Eq. (6.5) for \( T \) to find two solutions:

\[ T^a = -\frac{1}{2(1 + n)} \left( 1 + \sqrt{1 - \frac{4(1 + n)}{\delta S}} \right) \]

\[ T^b = -\frac{1}{2(1 + n)} \left( 1 - \sqrt{1 - \frac{4(1 + n)}{\delta S}} \right) \quad (6.7) \]

Recall that we restrict the analysis to interior equilibrium policies that do not feature full expropriation for any value of the state variable, \( \omega \). That is, we require \( T^0(\omega) < 1 \forall \omega \). To guarantee this condition for these two candidate solutions of \( T \), we need to impose the first order condition of the politicians (see Eq. (4.5)) evaluated at \( \omega = 0 \) to be negative when \( T = 1 \). Substituting \( \omega = 0 \) and \( T = 1 \) in Eq. (4.5), and imposing the expression to be negative yields the following inequality:

\[ \phi(1 + (1 + n)T)R > 1 \quad (6.9) \]

Let’s begin investigating the candidate solution \( T = T^a \). Substituting \( T^a \) in Eq. (6.9) yields the following inequality:

\[ \phi(R) \sqrt{1 - \frac{4(1 + n)}{\delta S} < \phi R - 2. \]

Clearly, for \( \phi < 2/R \), the above inequality is not satisfied, and thus \( T^a \) is not part of an interior equilibrium solution. For \( \phi > 2/R \), we can elaborate on the above expression to obtain the following inequality:

\[ \phi > \frac{2 \sqrt{\gamma(1 + n) + \delta S(1 + n)^2}}{\delta S - (1 + n)R^2} \]

Simple algebra shows that for \( \sigma^2 > R^2 > 1 \) (see Assumption 1), this inequality cannot hold for \( \phi > 2/R \). Hence, candidate solution \( T = T^a \) cannot be part of an interior equilibrium solution.

Let’s now turn to the candidate solution \( T = T^b \). Substituting \( T^b \) in Eq. (6.9) yields the following inequality:

\[ \phi(R) \sqrt{1 - \frac{4(1 + n)}{\delta S} > 2 - \phi R \}

Clearly, for \( \phi < 2/R \), the above inequality is always satisfied, and thus \( T^b \) can be part of an interior equilibrium solution. For \( \phi > 2/R \), the above inequality can be rewritten as \( \phi > \frac{2 \sqrt{\gamma(1 + n) + \delta S(1 + n)^2}}{\delta S - (1 + n)R^2} \). Notice that for \( \sigma^2 > R^2 > 1 \) (see Assumption 1), this inequality cannot hold for \( \phi > 2/R \). Hence, candidate solution \( T = T^b \) is part of an interior equilibrium solution if \( \phi > \frac{2 \sqrt{\gamma(1 + n) + \delta S(1 + n)^2}}{\delta S - (1 + n)R^2} \)

With \( T = T^b \), we can now solve the expression at Eq. (6.6) for \( \theta \). Simple algebra shows that \( \theta = \frac{2(\gamma - (1 + n) + \delta S(1 + n)^2)}{\delta S - (1 + n)R^2} \). Simple algebra shows that the denominator is always positive, while the numerator is positive for \( \phi < \frac{2 \sqrt{\gamma(1 + n) + \delta S(1 + n)^2}}{\delta S - (1 + n)R^2} \) Q.E.D.
Consider an increase in \(\gamma\). Additionally, squaring both terms in the above inequality, we get that for \(\phi > 1\) and \(\phi < 1\). While it is negative for \(\phi > 1\), and for \(\phi = 1\). Notice that \(\phi > 1\) i.e. for \(R > 2(1 + n)\). Two cases arise:

- for \(R > 2(1 + n)\), \(\theta + TR > 0\) if \(\phi \leq \frac{1}{2(1 + n)}\),
- for \(R > 2(1 + n)\), \(\theta + TR > 0\) if \(\phi \leq \frac{1}{2(1 + n)}\).

Simple algebra shows that the denominator of \(T\) being positive, i.e., \(\theta + TR > 0\) can be reduced to

\[
1 - \frac{4(1 + n)}{\phi R^2} > 0,
\]

It is easy to see that the inequality is always satisfied for \(R > 2(1 + n)\). Whereas, for \(R = 2(1 + n)\), \(1 + TR > 0\) if \(\phi > \frac{1}{2(1 + n)}\).

Hence, we have that:

- for \(R > 2(1 + n)\), the denominator is always positive, but the nominator is negative for \(\phi > 1\) (less than \(\frac{1}{2(1 + n)}\)). and hence \(T < 0\) for \(\phi = \phi^o\);
- for \(R = 2(1 + n)\), the denominator is positive for \(\phi > 1\), but the nominator is negative for \(\phi > 1\), and hence \(T < 0\) for \(\phi = \phi^o\).

Hence, \(T < 0\) for \(\phi = \phi^o\), Q.E.D.

**Proof of Proposition 4.4.** For \(\phi = \phi^o\), recall that the linear policy function is \(T^o(\alpha) = \theta + T^o(\omega)\) with \(\theta + T^o(\omega)\) defined in Proposition 4.2. Notice that we can write

\[
0 = \frac{1}{\phi} \left( \frac{2(1 - (1 + n))}{\phi(S - R(1 + n)) + \frac{1}{\phi}} Q - S - R(1 + n) \right) \gamma R - S.
\]  

(6.10)

Consider an increase in the average rate of return, \(R\), it is easy to see that

\[
\frac{\partial T^o}{\partial R} = \frac{2R}{\phi^2 \sqrt{1 + \frac{1}{\phi^2}}} > 0 \quad \text{and} \quad \frac{\partial^2 T^o}{\partial R^2} = \frac{\gamma R S^2}{(S - R(1 + n))^2} > 0,
\]

so, \(\gamma R S - \sigma^2 R\) imply that \(\gamma > 2R\).

Consider an increase in \(\phi\), it is easy to see that

\[
\frac{\partial T^o}{\partial \phi} = \frac{1}{\phi^2 S \sqrt{1 + \frac{1}{\phi^2}}} > 0.
\]

Notice that \(\theta\) can be written as

\[
0 = \frac{\phi SQ^2 + \gamma(1 - \phi Q)R}{\phi SQ^2 + (1 + n)(1 - \phi Q)R}.
\]

(6.11)

with \(Q = \frac{1}{2}(1 + \sqrt{1 - 4(1 + n)R})\) and \(\phi QR > 1\). Using the above expression, we have

\[
\frac{\partial^2 T}{\partial \phi^2} = -2 \left( \frac{\gamma(1 + n)}{(1 + n)^2} + \frac{S(1 + \sqrt{\Delta})}{\phi \sqrt{\phi^2}} \sqrt{\Delta} \right) < 0
\]

with \(\Delta = 1 - 4(1 + n)R\), Q.E.D.

**Proof of Proposition 5.1.** Comparing the first order condition respectively for the benevolent government (Eq. (3.3)) and for the politicians (Eq. (4.5)), we have that, in an interior equilibrium, the benevolent government and the politicians will adopt the same policy if \(\hat{\alpha} = \phi(1 + (1 + n)T)\). Recall that interior equilibrium policies, involving risk sharing at least when \(\omega = 0\), require respectively, \(\hat{\alpha} = \Lambda\) and \(\phi = \phi^o\). Using the expression for \(T^o\) at Proposition 4.2, the above expression can be rewritten as \(\phi = f(\hat{\alpha}) = \hat{\alpha} \left( \frac{1}{1 - \phi^o} - 1 \right)\).

Furthermore, it is trivial to see that for \(\phi\) and \(T^o(\alpha)\) that solve the benevolent government problem (for an interior equilibrium), if \(\alpha = f(\hat{\alpha})\), at \(T^o(\alpha) = T^o(\hat{\alpha})\) the first order condition of the politicians is negative, so that \(T^o(\alpha) = T^o(\hat{\alpha})\). And vice versa for \(\phi = f(\hat{\alpha})\).

**References**


Flora, P., 1887. Growth to Limits: The Western European Welfare States Since World War II. Volumes I to IV. Walter de Gruyter Inc.


