Early retirement and social security: A long term perspective*

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Abstract

We provide a long term perspective on the individual retirement behavior and on the future of retirement. In a political economic theoretical framework, in which incentives to retire early are embedded, we derive a political equilibrium with positive social security contribution rates and early retirement. Aging may have opposite (economic and political) effects on social security contributions and retirement decisions. For an actuarially "fair" social security system, we provide conditions for the political effect to dominate; in an "unfair" scheme, numerical simulations confirm a slight predominance of the political effect, as contribution rates increase. Economic slowdowns, captured by a reduction in the wages during the working period induce workers to postpone retirement, and – in the "unfair" system – lead to lower contribution rates. A reduction in the growth rate of the economy has opposite effects on the retirement decisions, leading – in the "unfair" system – to more early retirement.

Keywords: pensions, income effect, tax burden, politico-economic equilibrium

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1 Introduction

Retirement decisions represent one of the hottest issues in the current social security debate. Several studies (see Blondal and Scarpetta, 1998, and Gruber and Wise, 1999 and 2004) suggest that the individual retirement behavior is strongly affected by the design of the social security system. Retirement is concentrated at early and normal retirement age, as most individuals tend to retire as soon as this opportunity becomes available. Moreover, most social security systems provide strong incentives, such as large implicit taxes on continuing to work, to anticipate retirement. However, the individual retirement behavior is also largely influenced by wealth or income effects. Several recent studies (see Costa, 1988, Coronado and Perozek, 2003, Buetler et al.,2005, and Euwals et al., 2006) in fact show that both expected and unexpected increase in workers' income or wealth induce them to retire early.

The massive use of early retirement provisions and their generosity have contributed to the deterioration of the financial sustainability of the system, already under stress because of population aging. In fact, several international organizations - such as the European Union at the 2001 Lisbon Meetings - have advocated an increase in the effective retirement age, or - analogously - the increase in the activity rate among individuals aged above 55 years, as a key policy measure to control the rise in social security expenditure. In a nutshell, postponing the retirement age has become a common element to all social security reform's proposals. Yet, whether these policy prescriptions will actually be adopted depends on the politics of early retirement (see Fenge and Pestieau, 2005, for a detailed discussion of early retirement issues, and Galasso and Profeta, 2002, for a survey of the political economy of social security).

This paper provides a political economy framework to analyze the link between the retirement decisions and the political determination of the social security contribution rates. The design of the social security system – and in particular the contribution rate and the generosity of the pension benefits – affects the economic individual retirement decisions. In the political arena, our notion of Markov equilibrium relates the evolution of the social security system to the stock of capital – and hence to the individual savings. This politico economy model that characterizes the political equilibrium sequences of social security tax rates and the corresponding use of early retirement provisions suggests a non-trivial link between social security contributions and mass of early retirees, which depends mainly on the design of the social security system – in particular on its degree of redistributiveness – and on the level of income inquality in the society.

This paper's main contribution is to analyze the political role of (negative) income effects, for instance driven by aging or economic slowdowns, on the social security contribution rates and retirement age. In line with the existing literature (see Galasso and Profeta, 2002), our theoretical framework suggests that aging has two opposite effects on the contribution rates: it tends to decrease them, since it makes the public pension system less profitable, but it makes the median voter poorer, and thus induces higher social security. Despite this ambiguous result, however, aging may lead to an overall increase in the retirement age either because the direct effect due to the reduction in the generosity of the pension benefits overweights the indirect effect driven by the change in the contribution rate, or because both direct and indirect effects go in the same directions. Persistant economic slowdowns, as captured by a decrease of the growth rate of the economy or by a drop in the wages in the working period (i.e., in youth), induce a similar response. The social security contribution rate will typically be reduced, while the overall effect on the average retirement age will depend on the relative magnitude of direct effect – the reduction in the generosity of the pension benefits – and of the indirect effect – the change in the contribution rate. To fully characterize the analysis, we consider a (simpler) environment in which the social security system is actuarially "fair" at the margin, and an "unfair" system. In the former case, we are able to provide an analytical solution, while in the latter case we present a numerical example.

There exists a vast literature on retirement decisions. Already two decades ago, Feldstein (1974) and Boskin and Hurd (1978) analyzing the determinants of the decline in the labor force participation of elderly workers pointed at two key parameters of social security systems: the income guarantee and the implicit tax on earnings. Endogenous retirement decisions have been analyzed by showing how pension systems introduce distortions in the labor supply choice (see among others Diamond and Mirrless, 1978, Hu, 1979, Crawford and Lilien, 1981, and Michel and Pestieau, 1999). A new literature has lately emerged on the political economy of early retirement (see Fenge and Pestieau, 2005, Lacomba and Lagos, 2007, Casamatta et al., 2005, Cremer and Pestieau 2000, Cremer et al. 2004, Conde Ruiz and Galasso, 2003 and 2004), but has generally neglected the role of income effects. Politico-economic models of social security in a repeated voting environment have been studied by Cooley and Soares (1999), Galasso (1999), Boldrin and Rustichini (2000), Azariadis and Galasso (2002), Hassler et al (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005). These models however focus on social security and abstract from the role of retirement. They also neglect the political impact of income effects, which are instead empirically significant.

The paper is structured as follows. In the next section, we present a politico economic model. Section 3 analyzes the impact of aging and of a negative income effect on the steady state level of early retirement and social security. Section 4 concludes.

2 A Politico-Economic Model

2.1 The Economic Environment

We introduce a simple two -period overlapping generations model. Every period, two generations are alive, we call them young and old. Population grows at a non-negative rate, n. We consider a continuum of individuals heterogeneous in young and old wage income. The wage of a type-δ individual is $w_t^y = \delta \overline{w}_t^y$ in youth, and $w_t^o = \delta \overline{w}_t^o$ in old age, where $\overline{w}_t^y$ and $\overline{w}_t^o$ are respectively the average wage of young and old workers. Individual types δ are distributed according to some density function $f(\delta)$ over an interval $\left[\underline{\delta}, \overline{\delta}\right]$ with an average equal to 1 and cumulative density function $F(\delta)$.

Young individuals work: they receive a wage, w^y, pay a payroll tax, τ^y, on labour income and save all their disposable income for old age consumption. Savings are done through claims to capital, which yield in return r units of tomorrow's consumption. Old individuals decide what fraction, z, of the second period to spend working; in other words, they decide when to retire. An old individual who works a proportion z of the second period receives a net labor income equal to $w^o(1-\tau^o)$, for the fraction z of the period, and receives a pension p, which measures the overall pension transfer obtained in old age. The life time budget constraint for an agent born at time t is thus equal to:

$$c_{t+1}^o = \left(1-\tau_t^y\right) w_t^y \left(1+r\right) + \left(1-\tau_{t+1}^o\right) z_{t+1} w_{t+1}^o + p_{t+1} \tag{1}$$

where subscripts indicate the calendar time, so that c_{t+1}^o is old age consumption at time $t+1$, and τ_t^y and τ_{t+1}^o are the payroll taxes respectively paid by the young workers at time t and by the old workers at time $t+1$.

Agents maximize a logarithmic utility function, which depends on old age consumption and leisure:

$$U(c_{t+1}, z_{t+1}) = \ln c_{t+1}^o + \phi \ln(1-z_{t+1}) \tag{2}$$

where $\phi < 1$ measures the relative importance of leisure to the individuals.

The consumption good is produced using labor supplied by young and elderly workers and physical capital. We consider a linear production function

$$Y_t = \overline{w}_t^y L_t^y + \overline{w}_t^o L_t^o + r K_t \tag{3}$$

where Y_t is the production of the only consumption good at time t, L_t^y and L_t^o are the amount of labor by respectively young and elderly workers provided at time t, $\overline{w}_t^y$ and $\overline{w}_t^o$ are the respective average productivity, K_t is the stock of capital in the economy and r is the return on capital. Moreover, we assume that labor productivity grows at a rate g, so that $w_{t+1}^i = (1+g)\, w_t^i$ with $i = y, o$; and that the economy is dynamically efficient, $(1+r) > (1+n)(1+g)$.

Agents determine their retirement age, z_{t+1}, in order to maximize their utility at eq 2 subject to the budget constraint at eq 1. The solution of this maximization problems yields the following optimal individual labor supply decision:

$$\widehat{z}_{t+1} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi} \frac{(1-\tau_t^y) w_t^y (1+r) + p_{t+1}}{(1-\tau_{t+1}^o) w_{t+1}^o} \tag{4}$$

This individual retirement decision displays standard properties. A positive income effect, such as an increase in the net labor income in youth, induces all agents to retire early; while an increase in the net labor income in old age, or a decrease in the pension benefits, would lead them to postpone retirement – due mainly to a positive substitution effect. A generalized increase in wages –

both in youth and in old age – would instead combine income and substitutions effects, and leave the retirement age unaffected.

To ensure that no type-δ agent will end up either working the entire old age or retiring at the end of youth – that is, to avoid corner solutions in the individual labor supply decision – some conditions are to be imposed. In particular, with no social security system in place, i.e., for $\tau_t = 0\ \forall t$, no agent will ever want to work for the entire old age, and all agents will work for some period if $\phi < \overline{w}_t^o / \overline{w}_t^y (1+r)$. We shall hence assume that this condition holds. For positive contribution rates, the condition that individual labor supply decisions lead to interior solutions, i.e., $\widehat{z}_t \in (0,1)\ \forall t$, amounts to impose some restrictions on the dynamics of the contribution rates. We will return to these restrictions in the next section.

The labor supply of elderly individuals at time $t+1$, that is, the mass of employed elderly in the economy[1] can easily be obtained by aggregating all individuals' retirement decisions $L_{t+1}^o = \int_{\underline{\delta}}^{\overline{\delta}} \hat{z}_{t+1} f(\delta) d\delta$. Young workers have instead inelastic labor supply, and hence $L_t^y = 1$. Finally, capital market clearing requires the stock of capital to equalize aggregate savings. Hence, the per-capita stock of capital becomes:

$$K_{t+1} = \frac{1}{1+n} \int_{\underline{\delta}}^{\overline{\delta}} (1-\tau_t^y) \delta \bar{w}_t^y dF(\delta) = \frac{(1-\tau_t^y)\bar{w}_t^y}{1+n} \tag{5}$$

2.1.1 The Social Security System

We consider a defined benefit social secururity system. Every individual's pension benefit depends in part on her wage and in part on the average wage in the economy. This combination, which has extensively been used in this literature (see Casamatta et al.,2000, Conde Ruiz and Profeta, 2007), induces an element of within-cohort redistribution, from high to low income individuals. As in Tabellini (2000) and in Conde-Ruiz and Galasso (2005), this feature is crucial to ensure the political sustainability of social security system, through the support of the low ability young[2]. The pension benefit rule is:

$$p_{t+1} = \gamma \left(\alpha w_t^y + (1-\alpha) \overline{w}_t^y \right) \tag{6}$$

where p_{t+1} represents the pension received by a δ-type individual in old age, α determines the relative importance of the own wage in the benefit calculation, and thus defines the degree of redistributiveness of the social security system (the higher is α, the more Bismarkian, i.e., the less redistributive, the system is), and γ is a parameter that defines the overall generosity of the system. This last parameter is pinned down by the social security contribution rate through the budget constraint, as discussed below. Moreover, recall that $w_t^y = \delta \overline{w}_t^y$.

[1] Clearly, $1-Z$ defines the mass of (early) retirees.

[2] Alternatively, a 3-period OLG could be introduced in which social security is supported by a voting coalition of old and middle aged individuals. See Galasso and Profeta (2002) for a discussion of different elements leading to the political sustainability of social security.

Pension benefits are financed through the social security contributions. Hence total contributions, T_{t+1}, at time $t+1$ are equal to

$$T_{t+1} = (1+n)\,\tau^{y}_{t+1}\overline{w}^{y}_{t+1} + \tau^{o}_{t+1}\int_{\underline{\delta}}^{\overline{\delta}} z_{t+1} w^{o}_{t+1} dF(\delta) \tag{7}$$

where the terms on the right hand side represent the total contributions paid respectively by the young and the elderly workers at time $t+1$. We assume that the budget of this PAYG system is balanced every period, so that $T_{t+1} = \int_{\delta} p_{t+1} dF(\delta)$. Using equations 6 and 7, we obtain the following expressions for the generosity parameter:

$$\gamma = (1+n)\,\tau^{y}_{t+1}\frac{\overline{w}^{y}_{t+1}}{\overline{w}^{y}_{t}} + \tau^{o}_{t+1}\int_{\delta} z_{t+1}\frac{w^{o}_{t+1}}{\overline{w}^{y}_{t}} dF(\delta)$$

and hence for the pension benefit

$$p_{t+1} = \left((1+n)\,\tau^{y}_{t+1}(1+g) + \tau^{o}_{t+1}\int_{\delta} z_{t+1}\frac{w^{o}_{t+1}}{\overline{w}^{y}_{t}} dF(\delta)\right)\left(\alpha w^{y}_{t} + (1-\alpha)\,\overline{w}^{y}_{t}\right) \tag{8}$$

Pension benefits depend on the benefit rule at eq 6, which defines how redistributive these systems are, but their average generosity is determined by the growth rate of the economy – given by productivity and population growth – and (positively) by the labor supply of the elderly workers, which add extra contributions to the system. This last element is particularly relevant because the elderly labor supply decision depends also on the pension that the elderly expect to receive, as shown at eq 4.

2.1.2 Two Pension Systems

In the remaining of the paper, we will analyze two different pension arrangements. In the first scenario, which we call "fair", we consider a situation in which elderly workers do *not* contribute to the social security system, that is, $\tau^{o}_{t} = 0\ \forall t$. This pension system is actuarially fair at the margin, since elderly workers do not contribute, and the total amount of pension received in old age does not depend on the retirement age. This is equivalent to a situation in which individuals do pay contributions in their old age, $\tau^{o}_{t} > 0\ \forall t$, but their total pension benefits increases exactly by the amount of the contributions paid in old age (see Conde Ruiz et al., 2005). In the second scenario, which we call "unfair", we consider a case in which elderly workers do pay contributions, but these contributions do not bring any increase in their total pension benefits. Because of this, the system is actuarially *unfair* at the margin – hence providing an incentive to retire early.

<u>The "Fair" Pension System</u>

Setting the contributions of the elderly equal to zero, $\tau_t^o = 0\ \forall t$, greatly simplifies the analysis, and allows us to obtain a political equilibrium with a closed form solution (see section 2.2). The individual pension benefit of a δ-type agent at eq. 8 can be written as

$$p_{t+1} = \tau_{t+1}\left(1+n\right)\left(1+g\right)\overline{w}_t^y\left(\alpha\delta + \left(1-\alpha\right)\right) \tag{9}$$

with $\alpha = 1$ in a pure Bismarkian system and $\alpha = 0$ in a pure Beveridgean scheme.

With the above specification for the pension benefits, the optimal individual labor supply decision can be characterized as follows

$$\widehat{z}_{t+1} = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1-\tau_t^y)(1+r)\overline{w}_t^y}{\overline{w}_{t+1}^o} - \frac{\phi}{1+\phi}\frac{\tau_{t+1}^y\left(1+n\right)\left(1+g\right)\overline{w}_t^y\left(\alpha + \left(1-\alpha\right)/\delta\right)}{\overline{w}_{t+1}^o} \tag{10}$$

As this expression clearly shows, if individuals take into account the impact that an increase in wages has on their pension benefits, an overall raise in their wage at time t and $t+1$ leaves the retirement decision unaffected, since the income and substitution effects perfectly compensate one another. We will hence concentrate on changes in the wages in youth to analyze income effects and in the wages in old age to study incentive effects.

A sufficient condition for avoiding corner solution in the individual labor supply decision at old age, $\widehat{z}_t \in (0,1)\ \forall t$, for positive contribution rates, amounts to impose some restrictions on the dynamics of the contribution rates. In particular, we have that

$$\tau_{t+1} < \frac{\overline{w}_{t+1}^o - \overline{w}_t^y\left(1+r\right)\left(1-\tau_t\right)}{\left(1+n\right)\left(1+g\right)\overline{w}_t^y\left(\alpha + \left(1-\alpha\right)/\delta\right)}. \tag{11}$$

In this fair system, we can easily obtain a simple analitical expression for the mass of employed elderly in the economy at time $t+1$:

$$L_{t+1}^o = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1-\tau_t^y)(1+r)\overline{w}_t^y}{\overline{w}_{t+1}^o} - \frac{\phi}{1+\phi}\frac{\tau_{t+1}^y(1+n)\overline{w}_{t+1}^y\left(\alpha + \left(1-\alpha\right)\widehat{\delta}\right)}{\overline{w}_{t+1}^o} \tag{12}$$

with

$$\widehat{\delta} = \int_{\underline{\delta}}^{\overline{\delta}} \frac{1}{\delta} f(\delta) d\delta \tag{13}$$

Since individuals with different income display different retirement behaviors, the mass of retirees will depend on the distribution of income in the economy. In particular, due to the incentive effect embedded in the model, high income elderly workers will be induced to retire later than low income workers. A skewed income distribution hence tends to magnify the importance of the agents who enjoy very low income in old age and hence have an incentive to retire very early. The parameter $\widehat{\delta}$ captures this aspect by weighting the mass of these low-income elderly with their retirement behavior. The larger – for instance – the share of low-income elderly, the larger $\widehat{\delta}$ will be; and hence the larger the mass of (early) retirees $(1-Z)$.

Finally, by substituting the individual decision at eq.10 and the social security budget constraint, we can easily derive the indirect utility respectively of a type-δ young and old individual at time t, which we denote by $v_t^y(\tau_t, \tau_{t+1}, \delta)$ and $v_t^o(\tau_{t-1}, \tau_t, \delta)$.

The "Unfair" Pension System

In this scenario, all workers – young and old – pay the same contribution rate, $\tau_t^y = \tau_t^o = \tau_t \ \forall t$. As suggested by eq 4, the individual labor supply decision of the elderly will now depend both on the pension benefit and on the contribution rate imposed on the labor income in old age. A higher contribution rate reduces the net wage in old age, and thus induces more early retirement. This distortionary element introduces a Laffer curve, which was not present in the previous scenario. In turn, the pension benefit will now depend on the overall contributions by the elderly, and thus by the proportion of early retirees, as shown at eq.8. The following expressions summarize respectively the pension benefits for a δ-type retiree and the average labor supply by the elderly workers under this scenario:

$$p_{t+1} = \tau_{t+1}\left[(1+n)\bar{w}_{t+1}^y + G_{t+1}\right](\alpha\delta + 1 - \alpha) \tag{14}$$

$$L_{t+1}^o = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1-\tau_t)\bar{w}_t^y(1+r) + \tau_{t+1}\left[(1+n)\bar{w}_{t+1}^y + G_{t+1}\right](\alpha + (1-\alpha)\hat{\delta})}{(1-\tau_{t+1})\bar{w}_{t+1}^o} \tag{15a}$$

where $\widehat{\delta}$ is defined at eq.13 and

$$G_{t+1} = \int_\delta z_{t+1}\delta\bar{w}_{t+1}^o dF(\delta) = \frac{(1-\tau_{t+1})\bar{w}_{t+1}^o - \phi(1-\tau_t)\bar{w}_t^y(1+r) - \phi\tau_{t+1}(1+n)\bar{w}_{t+1}^y}{1+\phi-\tau_{t+1}} \tag{16}$$

represents the labor income of the elderly individuals at time $t+1$.

As in the previous case, individuals with different income choose different retirement behaviors, so that the overall mass of retirees will depend on the degree of income inequality in the economy, through the parameter $\widehat{\delta}$. Again, a larger value of the $\widehat{\delta}$, corresponding to a large share of low-income elderly, will be associated with more early retirees. A comparison of equations 9 and 14 shows instead the difference in terms of the distortionary effects of taxation. It is worth noticing that this "unfair" scenario allows to consider this distortionary effect, but at the cost of having to rely on a numerical solution.

Finally, we denote the indirect utility of a type-δ young and old individual at time t respectively by $v_t^y(\tau_t, \tau_{t+1}, \delta)$ and $v_t^o(\tau_{t-1}, \tau_t, \delta)$.

2.2 The Political Equilibrium

The purpose of this paper is to propose a theoretical framework in which to analyze the link between early retirement provision and the size of the social security system. As already discussed in the previous section, early retirement behavior may be induced by specific features of the social security system, such as the size of contribution rates and pension benefits. Here, we study the political determination of this social security contribution rate[3]. Every year, elections take place in which the current social security contribution rate is determined. All young and old agents participate at the elections. Their preferences over the contribution rate may differ – typically according to their income (δ type) and age. We follow a well established tradition in political economics by concentrating on the median voter decision. Moreover, due to the intergenerational nature of the system, we allow for some interdependence between current and future political decisions. In particular, we analyze Markov perfect equilibrium outcomes of a repeated voting game over the social security contribution rate. As costumary in this literature, we consider the state of the economy for the Markov equilibrium to be summarized by the stock of capital[4]. More specifically, at every period t, the median voter in each generation of voters – typically a young individual[5] – decides her most favorite social security system (i.e., the tax rate τ_t). In taking her decision, she expects her current decision to have an impact of future policies. In particular, her expectations about the future social security tax rate – and hence about her pension benefits – depend on the value of the state variable, i.e., on the stock of capital, according to a function $\tau_{t+1} = q^e(K_{t+1})$. Hence, future contribution rates depend on the stock of capital, which is in turn affected by the current voter's decision over the social security contribution rate, through its effect on the individual savings. The median voter's optimal decision can thus be obtained by maximizing her lifecycle utility with respect to τ_t, given expectations on the next period policy function $\tau_{t+1} = q^e(K_{t+1}) = Q(K_{t+1}(\tau_t))$:

$$\max_{\tau_t} v_t^y\left(\tau_t, \tau_{t+1,}\delta\right) = \max_{\tau_t} v_t^y\left(\tau_t, Q(K_{t+1}(\tau_t)), \delta\right) \tag{17}$$

We can now define our political equilibrium as follows

Definition 1 *A Markov political equilibrium is a pair of functions (Q, K), where $Q : [0, +\infty) \rightarrow [0, 1]$ is a policy rule, $\tau_t = Q(K_{t+1})$, and $K : [0, 1] \rightarrow [0, +\infty)$ is an aggregation of private decision rules, $K_{t+1}(\tau_t)$, such that the following functional equations hold:*

i) $Q(K_{t+1}) = \arg\max v_t^y\left(\tau_t, \tau_{t+1}, \delta^m\right)$ subject to $\tau_{t+1} = Q(K_{t+1}(\tau_t))$;

[3] It is important to notice that, depending on the scenario under analysis (namely whether we consider the "fair" or "unfair" system), the contribution rate, τ_t, may characterize different features. In the "fair" system, in fact, $\tau_t = \tau_t^y$ and $\tau_t^o = 0$ $\forall t$; while in the "unfair" case, $\tau_t = \tau_t^y = \tau_t^o$ $\forall t$. With this in mind, whenever it does not lead to confusion, we will hence drop the superscript and simply use τ_t.

[4] Examples of Markov equilibria in social security games are in Krusell et al.(1996), Grossman and Helpman (1998), Bassetto (1999), Azariadis and Galasso (2002), Hassler et al. (2003), Gonzalez-Eiras and Niepelt (2004), Forni (2005). Subgame perfect equilibrum outcomes of repeated games over social security have instead been analyzed by Boldrin and Rustichini (2000), Cooley and Soares (1999) and Galasso (1999) among many others.

[5] It is easy to show that, in this setting, every elderly voter will support a 100% contribution rate. For a positive population growth rate, $n > 0$, the median voter will hence be young.

ii) $K_{t+1}(\tau_t) = (1 - \tau_t^y)\bar{w}_t^y / (1+n)$*;*
iii) δ^m *identifies the median voter's type among the young.*

The first and last equilibrium conditions require that τ_t maximizes the objective function of the median voter – a type-δ^m young individual – taking into account that the future social security system tax rate, τ_{t+1}, depends on the current social security tax rate, τ_t, via its effect on the private savings, and thus on the stock of capital. Furthermore, it requires $Q(K_{t+1})$ to be a fixed point in the functional equation in part i) of the definition. In other words, if agents believe future benefits at any time $t+j$ to be set according to $\tau_{t+j} = Q(K_{t+j})$, then the same function $Q(K_{t+1})$ has to define the optimal voting decision today. The second equilibrium condition requires that all individuals choose their savings optimally.

In order to compute the political equilibrium, we have to consider the optimal social security tax rate chosen by the median voter at time t who maximizes the indirect utility function with respect to τ_t, given her expectations that $\tau_{t+1} = Q(K_{t+1}(\tau_t))$.

The corresponding first order condition is:

$$-\delta^m \overline{w}_t^y (1+r) - z_{t+1}^o \delta^m \overline{w}_{t+1}^o \frac{\partial \tau_{t+1}^o}{\partial \tau_t} + \frac{\partial p_{t+1}}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}}{\partial \tau_t} = 0 \tag{18}$$

where the first element represents the current cost to the median voter in terms of higher contributions, while the last term represents the future benefits corresponding to a higher pension, if a higher current contribution leads to a higher contribution rate also tomorrow: $\partial \tau_{t+1} / \partial \tau_t > 0$. In the "unfair" system, if $\partial \tau_{t+1}^o / \partial \tau_t > 0$, the median voter has to take into account the additional, future cost of paying higher contributions. With a redistributive design of the social security system, i.e., if $\alpha < 1$, we obtain the usual result that – with perfect financial markets – the most preferred contribution rate of a young individual is weakly decreasing in her income, since contributions depend on the wage income while – at least part of – the benefits do not. The elderly most preferred social security contribution rate does not depend on their type and is always larger than any young's. These features command the usual distribution of social security preferences among young and elderly voters.

It is now convenient to define the average return performance of the social security system relatively to the private claims to capital as $N = (1+n)(1+g)/(1+r)$. Clearly, the individual profitability of the social security system depends also on the individual type, δ, and on the degree of redistributiveness of the system, as measured by α.

The "Fair" Pension System

The next proposition characterizes the properties of the sequence of the equilibrium contribution rates in the "fair" social security scenario.

Proposition 2 *In a "fair" social security system (i.e., for* $\tau_t^o = 0 \ \forall t$*), if* $\delta^m < \frac{N(1-\alpha)}{1-N\alpha} (< 1)$ *there exists a Markov political equilibrium sequence of social security contribution rates* $\{\tau_t^*\}_{t=s}^{\infty} \in [0,1]$*, which evolves according to the* *following law of motion*

$$\tau^*_{t+1} = A - \frac{\delta^m (1-\tau^*_t)}{N(\alpha\delta^m + 1 - \alpha)},$$

where $A \in \left(\frac{1}{N\left[\alpha + \frac{1-\alpha}{\delta^m}\right]}, 1\right)$ *represents a free parameter pinned down by the first median voter's expectation of future policies, and* δ^m_t *is such that* $1 + (1+n_t)\, F(\delta^m_t) = 1 + n_t/2$. *This sequence converges to a non negative steady state:*

$$\overline{\tau} = \frac{AN(\alpha\delta^m + 1 - \alpha) - \delta^m}{N(\alpha\delta^m + 1 - \alpha) - \delta^m}.$$

Proof. See Appendix.

The above proposition suggests that – even in this dynamically efficient economy – a stable steady state with a positive level of the social security contribution rate may emerge as an equilibrium of the political game, if the individual return from social security to the median voter is larger than the return from private assets. In a Bismarkian system, i.e., when $\alpha = 1$, an equilibrium with a positive level of social security contributions hence fails to exist. However, with some redistribution, if the median voter is sufficiently poor, social security will be supported. For instance, in a pure Beveridgean system, i.e., with $\alpha = 0$, the median voter type has to be such that $\delta^m < N$. Hence, for a social security system to be in place, together with a highly redistributive social security system, the economy has to feature a high level of income inequality[6], as measured by the density function $f(\delta)$.

Unlike in most systems analyzed in this literature (see Conde-Ruiz and Galasso, 2003), here agents choose their retirement age. As shown at eq. 10, the social security contribution rates affect the individual retirement decisions and hence the overall use of the early retirement provisions. An increase in the contribution rate has two effects on the agents. It raises their contributions in youth, and increases their pension benefits. The former represents a negative income effect which induces individuals to retire later, while the latter effect calls for early retirement. Which of the two effects prevails depends on the average performance of the social security system relatively to the private assets, on its redistributiveness and on the individual income. Different individuals will typically have different responses to an increase in the contribution rate. If the system is sufficiently redistributive, low-income agents will anticipate their retirement, since the latter effect dominates, while high-income individuals will postpone it. The overall effect of an increase in income of the total mass of employed elderly will hence depend also on the distribution of ability in the society. For the fair system, the mass of employed elderly in the economy at a steady state is given by:

$$\overline{L}^o = \frac{1}{1+\phi} - \frac{\phi}{1+\phi}\frac{(1+r)\overline{w}^y}{\overline{w}^o}\left[1 + \overline{\tau}\left[N\left(\alpha + (1-\alpha)\,\widehat{\delta}\right) - 1\right]\right]. \qquad (19)$$

[6] A necessary but not sufficient condition is that the income distribution is skewed in the standard direction, and thus $\delta^m < E(\delta) = 1$.

It is easy to see that, if $N\left(\alpha+(1-\alpha)\widehat{\delta}\right)>1$, with $\widehat{\delta}$ defined at eq.13, an increase in the contribution rate reduces the overall employment among the elderly – i.e., it leads to more early retirement. This is because the (early retirement) effect induced among the low-income individuals is large enough to compensate for the increase in the retirement age of the high-ability types. This condition is more likely to hold the larger the share of low-income individuals (as measured by $\widehat{\delta}$), the more Beveridgean (α) and the more efficient (N), the system is. In fact, all these features characterize the impact of the contribution rate on the retirement decision of the low-income elderly. We will return to this crucial condition in the next section when investigating the impact of aging and growth on retirement in a "fair" social security system.

The "Unfair" Pension System

In the case of an "unfair" social security system, the political decision becomes more complex. First, an increase in tomorrow's contribution rate introduces an additional cost for today's median voter, as shown by the second term at eq. 18. Second, higher contributions have a non-linear effect on the pension benefits, because of the existence of a Laffer curve created by the effect of the contribution rate on the retirement decision (see section 2), as shown in the following expression:

$$\frac{\partial p_{t+1}}{\partial \tau_{t+1}}=\left[(1+n)\bar{w}_{t+1}^{y}+G_{t+1}+\frac{\partial G_{t+1}}{\partial \tau_{t+1}}\tau_{t+1}\right](\alpha\delta^{m}+1-\alpha) \tag{20}$$

where G_{t+1} represents the labor income of the elderly individuals at time $t+1$ (see eq. 16). We hence have to resort to a numerical solution to obtain the evolution of the political equilibrium social security contribution rates.

To parameterize this scenario, we consider that every period corresponds to 25 years. The average performance of the social security system is given by its average internal rate of return, which is measured by the product of the real wage growth rate and the population growth rate. We set them equal respectively to 2% and 1.5% annually. The performance of the alternative saving scheme – the claim to physical capital – is indicated by the annual real rate of return, which we set equal to 4.5%, in line with the average real return from the S&P Composite Index over the last hundred years. It follows that the average relative performance of the social security system with respect to this other saving scheme is equal to $N=(1+n)(1+g)/(1+r)=(1.015*1.02)^{25}/(1.045)^{25}=0.8$. In other words, social security pays out, on average, 20% less than private savings over the lifecycle.

To characterize the distribution of ability among the workers, we use the following cumulative Pareto distribution: $F(\delta)=1-\left(\frac{c}{\delta}\right)^{a}$ with $\delta\in(c,\infty\,]$ and $a>1$. In order to have that the average ability type is equal to one, we impose that $c=(a-1)/a$. To determine the skewness of the ability distribution, we choose $a=1.75$, which – given the population growth rate – delivers a median voter ability, $\delta^{m}=0.47$. The corresponding value of the parameter $\widehat{\delta}$, which weights the retirement behavior of the low-income elderly, is 1.48. Moreover, we set the average wage of young and old workers at time t to be equal, $\overline{w}_t^y = \overline{w}_t^o$. The parameter that measures the relative importance of leisure to the individuals.is set to $\phi = 0.1$, and the social security system is considered to be rather Beveridgean, with $\alpha = 0.25$. Finally, we set the constant of integration to $A = 0.25$.

As for the "fair" social security system, there exists a Markov political equilibrium sequence of social security contribution rates, which – for the parameter values specified above – converges to a steady state with a relatively low contribution rate: $\overline{\tau} = 12.6\%$. At this steady state, the average share of the old age worked is equal to $\overline{L} = 68.3\%$.

3 The future of Social Security and Early Retirement

3.1 The role of Aging

The equilibrium policy function obtained in the previous section for the "fair" social security system allows us to analyze the effects of aging on the social security tax rate and on the use of early retirement. In line with standard political economy models of social security (for a survey, see Galasso and Profeta, 2002), in our model, aging has opposite economic and political effects on the steady state social security tax rate. Aging reduces the profitability of the PAYG pension system with respect to alternative savings; and may convince the median voter to downsize the system – in order to increase her private provision of retirement income through alternative private assets. Yet, aging tends to change the identity of the median voter, who becomes poorer, and hence keener on increasing the contribution rate. Moreover, for a given contribution rate, an increase in the share of elderly in the population reduces the pension benefits, thereby inducing the elderly to postpone retirement.

Let $D = N\left(\alpha + (1-\alpha)\widehat{\delta}\right)$ characterize the impact of a change in the social security contribution on the low-income individuals' pension, and $\varepsilon_{\tau,n} = \frac{\partial \overline{\tau}}{\partial n}\frac{1+n}{\overline{\tau}}$ the elasticity of the equilibrium social security contribution to the population growth. The next proposition addresses the effect of aging on the social security contribution rate and on the overall employment among the elderly at steady state.

Proposition 3 *Consider a "fair" social security system (i.e., for $\tau_t^o = 0\ \forall t$). Aging (corresponding to a reduction in the population growth rate) decreases the steady state social security contribution rate, $\frac{\partial \overline{\tau}}{\partial n} > 0$, if $f(\delta^m) > \frac{1-\alpha}{2(1+n)(\alpha+(1-\alpha)\delta^m)\delta^m}$. Aging increases the steady state mass of employed elderly, $\frac{\partial \overline{L}}{\partial n} < 0$, if $D > 1$ and $\varepsilon_{\tau,n} > \frac{D}{1-D}$, or if $D < 1$ and $\varepsilon_{\tau,n} < \frac{D}{1-D}$.*

Proof. See Appendix.

The first part of this proposition summarizes the two effects of aging on social security discussed above. If the density function of the ability type around the median voter is sufficiently large, the impact of aging on the identity of the median voter will only be marginal. The political effect will hence be relatively small, and the economic effect will dominate. Aging will then lead to a reduction in the social security contribution rate, being mainly driven by the economic effect[7]. The second part of the proposition studies the effect of aging on the retirement decisions. Aging has a direct impact on the incentive to retire: by reducing the pension benefit – for a given contribution rate – it always makes early retirement less appealing. But aging will also have an indirect effect, which goes through the change that it induces in the contribution rate. This indirect effect may go in opposite directions. This proposition characterizes the two circumstances under which aging leads to postponing retirement. First, consider the case in which $N\left(\alpha + (1-\alpha)\widehat{\delta}\right) > 1$, that is, an increase in the contribution rate reduces the overall employment among the elderly, because it induces a large effect among the low-income individuals. If aging leads to a reduction in the contribution rate, than both the direct and the indirect effects will lead in the same direction: postponing the overall retirement age. But, even if the contribution rate increases – thereby leading to a reduction in the overall employment among the elderly, aging may still lead to an increase in the overall retirement age, provided that the direct effect dominates. The above proposition summarizes this condition in terms of the elasticity of the contribution rate with respect to a change in the population growth rate. Second, consider that $N\left(\alpha + (1-\alpha)\widehat{\delta}\right) < 1$, i.e., higher contribution rates now increase the overall employment among the elderly, because the effect is larger among the high-income individuals. In this case, if aging implies a higher contribution rate, the direct and indirect effects will both lead to postponing the average retirement age. Yet, even if the contribution rate drops, aging may still be associated with later overall retirement, if the direct effect prevails..

The "Unfair" Pension System

How does aging affect the social security contribution rate and the overall employment among the elderly at steady state in an "unfair" pension system? Unlike in the previous case, we can not provide an analitical answer and we have to rely on a numerical characterization.

Table 1 summarizes the effects of a change in the population growth rate on the social security contribution rate and on the retirement age. Population aging – defined as a reduction in the population growth rate – lead to higher contribution rates. In this numerical example, the political effect thus prevails, albeit slightly, as suggested by the change in the median voter's ability type, induced by the increased share of the elderly. Aging brings also a slight increase in the average age of retirement. This is in line with the existence of a direct effect of aging on retiurement decisions, due to the reduction of the pension

[7] Which effect will dominate represents an empirical question that remains to be settled. For instance, Galasso and Profeta (2004) simulate the political effect to prevail, whereas Razin's et al (2002) empirical analysis leads to the opposite results. See also Disney (2007), Simonovits (2007) and Galasso and Profeta (2007) for empirical and theoretical contributions on this debate.

benefits. The indirect effect associated with the higher contribution rate partially moderates the direct impact. Overall, the retirement age increases only marginally.

3.2 The Role of the Income effects

In this section we highlight the role of income effects on retirement decisions and thus on the social security equilibrium tax rate. While most studies concentrate on the role of the incentives (substitution effect) in the retirement behavior, available empirical evidence (see for instance Costa, 1998, and Coronado and Perozek, 2003) suggest that income effects do play a crucial role in the labor supply decisions of elderly workers.

The political economy model presented in section 2 may help to understand how changes in the individual retirement decisions induced by income effects modify the political determination of social security and hence the equilibrium mass of early retirees. To analyze the role of the income effect, we consider two different experiments. First, we examine a permanent variation in the growth rate of the economy, which affects the average return of the social security system, but also the future wage income – and thus the opportunity cost of retiring. Second, we analyze the pure income effect due to a drop in the wages at youth, which entails no substitution effect[8].

A drop in the growth rate will hence induce two effects on the retirement decisions: a direct negative impact due to lower pension benefits and an indirect effect driven by the change in the contribution rate, and in the wage in old age. A reduction in the wages in youth reduces the individual net income, but also her pension benefits. The next proposition summarizes the effects on the overall employment among the elderly at steady state, in a "fair" social security system.

Proposition 4 *Consider a "fair" social security system (i.e., for $\tau_t^o = 0\ \forall t$). A reduction in the growth rate of the economy (g) increases the steady state mass of employed elderly, $\frac{\partial \overline{L}}{\partial g} < 0$, if $D > 1$ and $\varepsilon_{g,n} > \frac{D}{1-D}$, or if $D < 1$ and $\varepsilon_{g,n} < \frac{D}{1-D}$. A reduction in the wage in youth ($\overline{w}^y$) increases the steady state mass of employed elderly, $\frac{\partial \overline{L}}{\partial \overline{w}^y} < 0$.*

Proof. See Appendix.

A reduction in the growth rate of the economy has a direct impact on the incentive to retire: by reducing the pension benefit – for a given contribution rate – it always makes early retirement less appealing. But if the economy slows down, there will also be an indirect effect, through the contribution rates. This indirect effect may go in opposite directions. This proposition suggests that economic slowdowns may lead to later retirement in two circumstances. As already discussed, when $N\left(\alpha + (1-\alpha)\widehat{\delta}\right) > 1$, an increase in the contribution rate reduces the overall employment among the elderly. If lower economic growth reduces the contribution rate, than the overall retirement age will certainly be

[8] Indeed, the pension benefits are affected since the tax base shrinks.

postponed. But even when the contribution rate increases, the overall retirement age may still increase, provided that the direct effect dominates. Whether this will occur depends on the the elasticity of the contribution rate with respect to a change in the growth rate of the economy. If instead $N\left(\alpha+(1-\alpha)\widehat{\delta}\right)<1$, higher contribution rates increase the overall employment among the elderly. In this case, the direct and indirect effects will both lead to postponing the average retirement age if economic slowdowns implies a higher contribution rate. Yet, this drop in economic growth may still be associated with later overall retirement even if the contribution rate drops, if the direct effect prevails. A reduction in the wage rate in youth has a simpler effect. Being poorer – due to lower income in youth – and facing lower pension benefits, individuals choose to retire earlier.

In an "unfair" system, a reduction in the growth rate or in the wage in youth has additional effects through the Laffer curve induced by the taxation on the elderly workers. The effects of a change in the growth rate are summarized at table 2. In line with the existing literature (see Galasso, 1999), an increase in the growth rate of the economy leads to more social security contributions, since the pension system becomes more efficient. Yet, despite the more generous pensions, the average retirement age increases, due to the contemporaneous raise in the old age wage, which represents the opportunity cost of retiring. This numerical exercise thus suggests that lower growth could lead to lower pensions, lower wages and yet to early retirement. If instead a reduction in the wage in youth is considered, the tax base shrinks – thus inducing lower contribution rates, and individuals react to the lower lifetime income by working longer years. As shown in table 3, in fact, the steady state mass of elderly workers increases.

4 Conclusions

This paper concentrates on the long term determinants of the retirement decisions and on the future evolution of social security system and early retirement provisions. In our politico-economic environment, every period a young low-income median voter determines the social security contribution by considering the evolution of the early retirement behavior. We emphasize the role of substitution and income effects in these retirement decisions. The incentive effects have been analyzed by a large empirical literature, which shows how (at the margin) non-actuarially fair pension systems may induce rational agents to retire early, by reducing the opportunity cost of leisure. Income effects have instead largely been neglected in models of retirement and social security, despite the empirical evidence suggesting that variation in income and wealth do modify individual retirement decisions.

In line with the existing political economy literature (see Galasso and Profeta, 2002), in our model, aging has opposite economic and political effects on social security. We provide conditions for either effect to dominate in a "fair" social security system. Numerical simulations for an "unfair" social security system suggest that the political effect slightly prevails and contribution rate increase.

Yet, aging may lead to a reduction in the widespread use of early retirement provisions. By commanding less generous pension benefits (for a given level of contribution rates), aging induces workers to postpone retirement. If the political effect is not so overwelming as to determine a sizable increase in social security contributions, and thus also in pension benefits, at steady state aging societies will be associated with less early retirement. Simulations for an "unfair" social security system confirm these findings.

Economic conditions also matter for social security determination. Our model suggests that a decrease in the wage income in youth leads to fewer early retirees. The numerical simulation shows that social security tax rate may decrease. To the extent that this change in young wage income may proxy for a drop in the life-time labor income, this may prove a crucial result to understand the future evolution of the early retirement provision. Societies characterized by economic stagnation or raise in lifetime inequality that increase the share of low-income individuals may thus be associated with a less pervasive use of these early retirement provisions. Yet, if we concentrate on reduction in the economic growth rate, results are mixed. We provide conditions for lower growth to lead to higher labor participation rates among the elderly in a "fair" system. However, numerical simulations of the "unfair" system show that lower growth is associated with lower tax rates and more early retirement.

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5 Appendix

5.1 Proof of proposition 2

Using eq. 9, the first order condition of the median voter at eq. 18 can be written as:

$$-\delta_t^m \overline{w}_t^y (1+r) + (1+n)(1+g)(\alpha\delta^m + (1-\alpha))\,\overline{w}_{t+1}^y \frac{\partial \tau_{t+1}}{\partial \tau_t} = 0 \tag{21}$$

where, given the median voter's expectations on the next median voter's behavior,

$$\frac{\partial \tau_{t+1}}{\partial \tau_t} = Q' \frac{\partial K_{t+1}}{\partial \tau_t} \tag{22}$$

with

$$Q' = \frac{\partial Q}{\partial K_{t+1}} \tag{23}$$

and

$$\frac{\partial K_{t+1}}{\partial \tau_t} = \frac{-\bar{w}_t^y}{1+n} \tag{24}$$

Using the above equations, we obtain

$$Q' = -\frac{\delta_t^m (1+r)}{(1+g)\,\overline{w}_t^y\,(\alpha\delta^m + (1-\alpha))} \tag{25}$$

Integrating the above equation with respect to K_{t+1} we obtain

$$\tau_{t+1} = Q(K_{t+1}) = A - \frac{\delta_t^m (1+r)}{(1+g)\,\overline{w}_t^y\,(\alpha\delta^m + (1-\alpha))} K_{t+1} \tag{26}$$

where A is a constant of integration.

Using eq. 5, eq. 26 can be written as

$$\tau_{t+1} = A - \frac{\delta^m (1-\tau_t)}{N(\alpha\delta^m + 1 - \alpha)}$$

where $N = (1+n)(1+g)/(1+r)$. It is easy to see that this linear law of motion features non-negative social security contribution rates converging to a non-negative steady state if $A - \frac{1}{N\left[\alpha + \frac{1-\alpha}{\delta^m}\right]} > 0$ and $\frac{1}{N\left[\alpha + \frac{1-\alpha}{\delta^m}\right]} < 1$. Furthermore, the steady state value of the contribution rate is less than 1 if $A < 1$.

Finally, to determine the identity of the median voter, notice that – by equation 21 – the most preferred social security contribution rate among the young is weakly decreasing in their income; and that the old always command a higher tax rate than the any young. For non-negative population growth rates, the median voter is among the young and has a type δ_t^m, which divides the distribution of preference in halves: $1 + (1+n_t)\,F(\delta_t^m) = 1 + n_t/2$.

5.2 Proof of proposition 3

Consider the steady state social security contribution rates at proposition 2. It is easy to see that – for a given median voter type, δ_t^m – aging reduces the contribution rate. In fact

$$\frac{\partial\overline{\tau}}{\partial n}_{\delta^m} = \frac{(1-A)\,\delta^m\,(\alpha\delta^m+1-\alpha)}{[N\,(\alpha\delta^m+1-\alpha)-\delta^m]^2}\frac{1+g}{1+r} > 0.$$

Yet, aging affect also the median voter type, since $\frac{\partial\delta^m}{\partial n} = \frac{1}{2(1+n)^2 f(\delta^m)}$. Moreover, a change in the median voter type induces the following poltical effect on the contribution rate:

$$\frac{\partial\overline{\tau}}{\partial\delta^m} = -\frac{N\,(1-A)\,(1-\alpha)}{[N\,(\alpha\delta^m+1-\alpha)-\delta^m]^2} < 0.$$

Hence, the overall effect of aging on the steady state social security contribution rate, when also the change in the median voter type is considered becomes

$$\frac{\partial\overline{\tau}}{\partial n} = \frac{(1-A)\,N}{(1+n)\,[N\,(\alpha\delta^m+1-\alpha)-\delta^m]^2}\left\{\delta^m\,(\alpha\delta^m+1-\alpha) - \frac{1-\alpha}{2\,(1+n)^2\,f\,(\delta^m)}\right\}$$

so that the sign of the overall effect will depend on the term in parentesys.

Consider now the steady state mass of employed elderly at eq. 19. Aging induces a direct effect on this overall retirement age (see the first term in the equation below) and an indirect effect, through the changes in the contribution rate (second term):

$$\begin{aligned}\frac{\partial\overline{L}^o}{\partial n} &= -\frac{\phi}{1+\phi}\frac{\overline{\tau}(1+g)\overline{w}^y}{\overline{w}^o}\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)+\\ &\quad+\frac{\partial\overline{\tau}}{\partial n}\frac{\phi}{1+\phi}\frac{\overline{w}^y}{\overline{w}^o}\left[1+r-(1+n)\,(1+g)\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]\end{aligned}$$

which can more conveniently be written as

$$\frac{\partial\overline{L}^o}{\partial n} = \frac{\phi}{1+\phi}\frac{\overline{\tau}(1+r)\overline{w}^y}{(1+n)\,\overline{w}^o}\left[\varepsilon_{\tau,n}\left[1-N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]-N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]$$

with $\varepsilon_{\tau,n} = \frac{\partial\overline{\tau}}{\partial n}\frac{1+n}{\overline{\tau}}$. Hence, it is straightforward to see that for $N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right) > 1$, $\frac{\partial\overline{L}}{\partial n} < 0$, if $\varepsilon_{\tau,n} > \frac{N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}{1-N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}$. For $N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right) < 1$, instead $\frac{\partial\overline{L}}{\partial n} < 0$, if $\varepsilon_{\tau,n} < \frac{N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}{1-N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}$.

5.3 Proof of proposition 4

Consider the steady state social security contribution rates at proposition 2. It is easy to see that economic slowdowns reduces the contribution rate. In fact

$$\frac{\partial \overline{\tau}}{\partial g}=\frac{(1-A)\,\delta^{m}\,(\alpha\delta^{m}+1-\alpha)}{\left[N\,(\alpha\delta^{m}+1-\alpha)-\delta^{m}\right]^{2}}\frac{1+n}{1+r}>0.$$

Consider now the steady state mass of employed elderly at eq. 19. An economic slowdown induces a direct effect on this overall retirement age (see the first term in the equation below) and an indirect effect, through the changes in the contribution rate (second term):

$$\begin{aligned}\frac{\partial \overline{L}^{o}}{\partial g} &= -\frac{\phi}{1+\phi}\frac{\overline{\tau}(1+n)\overline{w}^{y}}{\overline{w}^{o}}\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)+ \\ &\quad +\frac{\partial \overline{\tau}}{\partial g}\frac{\phi}{1+\phi}\frac{\overline{w}^{y}}{\overline{w}^{o}}\left[1+r-(1+n)\,(1+g)\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]\end{aligned}$$

which can more conveniently be written as

$$\frac{\partial \overline{L}^{o}}{\partial n}=\frac{\phi}{1+\phi}\frac{\overline{\tau}(1+r)\overline{w}^{y}}{(1+g)\,\overline{w}^{o}}\left[\varepsilon_{\tau,g}\left[1-N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]-N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)\right]$$

with $\varepsilon_{\tau,g}=\frac{\partial \overline{\tau}}{\partial g}\frac{1+g}{\overline{\tau}}$. Hence, it is straightforward to see that for $N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)>1$, $\frac{\partial \overline{L}}{\partial g}<0$, if $\varepsilon_{\tau,g}>\frac{N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}{1-N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}$. For $N\left(\alpha+(1-\alpha)\,\widehat{\delta}\right)<1$, instead $\frac{\partial \overline{L}}{\partial g}<0$, if $\varepsilon_{\tau,g}<\frac{N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}{1-N\left(\alpha+(1-\alpha)\widehat{\delta}\right)}$. Finally, it is easy to see that $\frac{\partial \overline{L}}{\partial \overline{w}^{y}}<0$.

Table 1

n	τ	L	δ^M
1	12.9	68.5	0.458
1.25	12.8	68.4	0.465
1.5	12.6	68.3	0.472
1.75	12.5	68.2	0.479
2	12.3	68.1	0.485

Table 2

g	τ	L	P	$(1-\tau)w$
1.5	11.8	66.2	31.5%	1.856
1.75	12.2	67.3		
2	12.6	68.3	38.6%	2.08
2.25	13	69.3		
2.5	13.4	70.1	46.8%	2.329

Table 3
ϕ=0.1, α=0.25, α=1.75,A=0.25,g=2%,n=1.5%

y_y	y_{old}	τ	L	P
0.9	1	12.1	70.6	34.9%
0.95	1	12.4	69.4	36.7%
1	1	12.6	68.3	38.6%
1.05	1	12.9	67.2	40.4%
1.1	1	13.1	66	42.2%

Figure 1: