SHORT TERM VOLATILITY TIMING REDUCES DOWNSIDE RISK

Giovanni Barone Adesi (corresponding author)
Institute of Finance, University of Southern Switzerland,
Via Buffi 13, CH-6900 Lugano,
tel: +41 91 912 47 53, fax: +41 91 912 46 47,
e-mail: Giovanni.Barone@lu.unisi.ch

Patrick Gagliardini
Institute of Finance, University of Southern Switzerland,
Via Buffi 13, CH-6900 Lugano,
e-mail: Patrick.Gagliardini@lu.unisi.ch

Fabio Trojani
Institute of Finance, University of Southern Switzerland,
Via Buffi 13, CH-6900 Lugano,
e-mail: Fabio.Trojani@lu.unisi.ch

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Abstract

We analyze the short term trade-off between market return and market risk by comparing a class of straightforward dynamic index "switching" strategies. Because of the pronounced non-normality of index returns at short horizons we measure risk adjusted performances by integrating standard variance based risk measures with indicators of downside risk like VaR and Expected Shortfall. Comparisons are carried out both through parametric simulation and nonparametric bootstrap in order to account for estimation and model risk. We find a significant contribution of volatility rules in enhancing the trade-off between return and downside risk. On the other hand, empirical differences observed using mean variance criteria are not sufficient alone to motivate making use of volatility timing. Moreover, the empirical strategy rankings obtained by mean variance in this setting can be paradoxical. Indeed, they often prefer "fixed-weights" pay-offs that are stochastically dominated by the end of period pay-off of a dynamic strategy. We conclude that volatility strategies have important economic value for investors and portfolio managers that are concerned with the downside risk of their dynamic portfolios.

JEL Classification: G0, G1

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I Introduction

Numerous studies have investigated to which extent state variables determining the opportunity set underlying an investor's actions can be predicted. This is a key issue for dynamic asset allocation since predictability implies dynamic optimal portfolio rules that are generally intrinsically different from the solution implied by a pure i.i.d. setting[1]. Indeed, when returns are predictable demand for risky assets is determined essentially by two components/motives: a conditionally optimal mean variance allocation and a further intertemporal hedging position[2], as in standard intertemporal asset pricing models à la Merton (1969), (1971). In these models optimal allocations are completely characterized by two ingredients: investor's preferences (obviously) and the first two conditional moments of the joint stochastic process of asset returns and some further (possibly latent) risk factors. Since there is now ample evidence in the literature that means, variances and covariances of assets returns can be predicted to some extent[3], investigating the impact of this predictability on the performance of non-trivial dynamic strategies (compared, for instance, to fix-weights trading rules) is an important research area. It contributes first to our general understanding of dynamic portfolio management, second to an objective evaluation of the feasibility of such strategies for practical management purposes and third to the development of more adequate (dynamic) benchmarks for performance evaluation.

The economic relevance of returns predictability for dynamic asset allocation had been investigated already by a few authors. Models where a significant economic contribution of predictability in expected returns is documented are investigated by Brennan, Schwartz and Lagnado (1997) and Bielecki, Pliska and Sherris (2000). In two related papers Barberis (2000) and Xia (2001) observe that predictability and predictability coupled with learning, respectively, generate important horizon effects when linked to different forms of parameter uncertainty. Finally, unconditional models of performance evaluation that incorporate the conditioning information contained in some popular predictors of expected returns show that most investment funds cannot beat such a dynamic benchmark; cf. Bansal and Harvey (1996)[4]. Some research results on the effectivenes of some simple "switching" market volatility strategies has been produced by Graham and Harvey (1996) and Copeland and Copeland (1999). On the other hand, Busse (1999) examines the trading behaviour of active portfolio managers and documents a negative relation between the market exposure of many investment funds and predicted volatility[5]. In a related paper, Fleming, Kirby and Ostdiek (2000), investigate systematically the economic value of volatility timing for (very) short term asset allocation strategies. Using mean variance based performance measures and an investment universe composed by stocks, bonds and gold they find that volatility timing is more effective than a "static" (fix weights) investment strategy[6].

In this paper we focus on the effectiveness of daily volatility strategies from the perspective of a short-term index investor (having investment horizons between 1 and 12 monthes, say) who is assumed to be concerned with at least two relevant risk dimensions: volatility and downside risk. This approach is partly motivated by the pronounced non-normality of market returns at short horizons, which can lead to strongly biased risk assessments when using only variances or related risk measures. In fact, stochastic volatility is a feature causing unconditional non-normality even for conditionally normal returns[7]. It seems therefore natural to measure the effectiveness of volatility strategies also by their empirical ability/failure to change unconditional non-normality in the returns of dynamic portfolios. Further, downside risk is a risk dimension that is today largely known and taken into account by institutional investors in the praxis, where risk measures like Value-at-Risk (VaR) are often "the" official measure of risk for many risk management and risk controlling purposes. Finally, from a more theoretical perspective, downside risk and loss aversion (cf. Tversky and Kahneman (1992)) can be expected to be more relevant at short horizons than at long horizons since at short horizons large losses on risky securities are realized more often (see also Benartzi and Thaler (1995) and Barberis, Huang and Santos (1999)).

Looking behind the statistical problem of predicting volatility it is difficult to decide which further selection or combination of predictive variables (if any) the investor should focus on at short horizons. It is intuitively obvious that different objective functions place different emphases on the various features of the conditional return distribution. In the mean-variance paradigm the first order condition for optimization implies that we should look at variables best predicting the ratio of conditionally expected returns to variances. However, at daily frequencies expected returns are well known to be extremely hard to predict precisely (Merton (1980)). No economic variable acting as a significant predictor of expected returns at short horizons has been really identified in the literature[8]. Further, while standard equilibrium considerations suggest a positive (possibly linear) relation between conditionally expected returns and variances the statistical evidence for this is sparse. Indeed, while at monthly frequencies Campbell (1987), Glosten, Jagannathan and Runkle (1993) and Whitelaw (1994) observe market volatility changing with information variables such as interest rates, most of the evidence reported produced a negative or insignificant relation from volatility[9] to monthly expected returns. On the other side, Li (1998) finds monthly stock market risk premia to be related to the variance of further economic variables but not to their own variance. At daily frequencies, the link between expected returns and volatilities is even more elusive.

Rather than trying to identify the underlying relation between (conditional) expected returns and volatilities with a single model we therefore estimate some simple GARCH models (Bollerslev (1986)) under the simplest possible hypothesis on the functional form of the relation between returns and volatilities: constant expected returns. We make then use of the obtained GARCH parameter estimates to construct a set of straightforward market volatility timing rules for the S&P500, the Dow Jones, the FTSE100 and the Nikkei indices. These strategies are analyzed with respect to their effectiveness in enhancing the unconditional trade-off between risk and expected return, when risk is measured both by volatility and a measure of downside risk like VaR and Expected Shortfall (Artzner et al. (1998)). Similarly to previous studies, we control for the impact of estimation and model risk by making extensive use of both parametric Monte Carlo simulation and nonparametric bootstrap (Efron (1979) and Künsch (1989)).

We limit our analysis to simple "switching" index strategies basically for the following reasons. First, including more than one risky asset would possibly require a multifactor benchmark model in order to quantify the variance/covariance risk of a dynamic strategy realistically (see for instance Scruggs (1998)). Second, in a multi-assets framework measures of downside risk like VaR have difficulties aggregating individual risk (even for risks that are cross-sectionally independent) making the empirical distinction between volatility and downside risk less clear-cut, primarily because of some counter-intuitive effects of portfolio diversification (cf. again Artzner et al. (1998)).

Our results indicate that volatility strategies have value in ameliorating the risk-return trade-off of short term investors: they systematically reduce the empirically observed end of period downside risk. On the other hand, no such consistent differences between dynamic and fix weight strategies are found when risk is measured by variances or volatilities. In fact, by pure mean variance criteria we observe often paradoxical strategy rankings where first order stochastically dominated pay-offs are selected. Such counter-intuitive conclusions disappear when downside risk is taken into account. This evidence suggests that volatility timing and related strategies are particularly effective when used as a risk - management tool rather that as a pure asset allocation instrument.

In Section 2 we describe our methodology for assessing the effectiveness of volatility strategies in enhancing the trade-off between market return and market risk. Section 3 presents the empirical results of our analysis while Section 4 concludes with some final remarks and hints for further research.

II Methodology

Let r_t and r_{ft} be the return at time t of a stock index and of a conditionally riskless asset, respectively, and

$$\mu_t := E_{t-1}(r_t) := E(r_t|I_{t-1}) \quad , \quad \sigma_t^2 := Var_{t-1}(r_t) := Var(r_t|I_{t-1}) \quad , \tag{1}$$

be the time $t-1$ conditional expectation and conditional variance, respectively, of index returns.

We consider a short term investor investing only in the index and in the conditionally riskless asset. Our manager is interested in the *unconditional* performance of a trading rule $(w_t)_{t=0,..,H-1}$ over the given holding period $[0, H]$, where w_t is the fraction of current wealth she is ready to invest in the risky asset at time $t-1$, depending on the available market information I_{t-1}. We are interested in quantifying the effectiveness of dynamic strategies exploiting volatility predictability and to find evidence of an enhanced trade-off between expected returns and risk as measured by volatility and (or) downside risk.

II.A Optimizing Return vs. Volatility

Several dynamic portfolios $(w_t)_{t=0,..,H-1}$ are feasible, from a simple fix weight rule holding fixed proportions $w_t = w$, for $t = 0, .., H-1$, to more sophisticated strategies attempting to exploit the kind of information available at time $t-1$ given a market structure and an optimizing utility criterion. We limit our analysis to standard mean variance based volatility strategies.

The optimal choice of a conditional mean variance optimizer solving the conditional optimizing problem

$$\max_{w_t} \left[w_t \mu_t + (1 - w_t) r_{ft} - \frac{\lambda}{2} w_t^2 \sigma_t^2 \right] \tag{2}$$

is [10]:

$$w_t = \frac{1}{\lambda} \cdot \frac{\mu_t - r_{ft}}{\sigma_t^2} \tag{3}$$

where λ is a constant that can be interpreted as a risk aversion parameter with respect to variance risk. Hence, all what is needed by a mean variance investor to behave conditionally optimally is a prediction of the reward to variance ratio $\frac{\mu_t - r_{ft}}{\sigma_t^2}$.

Notice that the allocation (3) is optimal with respect to a corresponding conditional risk measure, but not generally with respect to unconditional ones. The optimal rule of an investor solving the unconditional version of the optimizing problem (2):

$$\max_{(w_t)} E\left(w_t \mu_t + (1 - w_t) r_{ft} \right) - \frac{\lambda}{2} Var\left[w_t \mu_t + (1 - w_t) r_{ft} \right] \quad , \tag{4}$$

is of the form (cf. Ferson and Siegel (1999)):

$$w_t = \frac{1}{k(\lambda)} \cdot \frac{\mu_t - r_{ft}}{\sigma_t^2 + (\mu_t - r_{ft})^2} \quad , \tag{5}$$

with a λ-dependent constant $k(\lambda)$. In this case it can be readily verified that the estimated numerical values of the optimal strategies implied by a conditional and an unconditional mean variance criterium (as a function of σ_t) are virtually indistinguishable in our empirical analysis below. Hence, we can expect the unconditional performance observed for the volatility strategy (3) to attain appoximately the optimum in terms of unconditional mean variance criteria, at least over investment horizons of one day[11].

II.B Predicting Conditional Means and Variances

The practical implementation of the optimal stategy in the last section needs a model for predicting reward to variance ratios of excess index returns. This can be achieved by estimating a parametric model of the time behaviour of the first two conditional moments of index returns[12].

We consider some well-known parametric models for the conditional distribution of $r_t - r_{ft}$, where conditional means and variances are parameterized by some corresponding functions m_t and h_t evaluated at an unknown parameter vector $\vartheta = \left(\alpha', \beta'\right)'$:

$$\mu_t - r_{ft} = m_t(\vartheta) \quad , \quad \sigma_t^2 = h_t(\vartheta) \quad . \tag{6}$$

A huge number of articles has investigated the predictive power of parametric volatility models[13] of the general form (6), finding that volatility is to some extent predictable across a wide range of assets and using different volatility specifications. While the explanatory power of these models is typically low, based on standard volatility measures, Andersen and Bollerslev (1998) show that GARCH models explain about 50% of the variation in ex-post volatility, measured by cumulative squared intraday returns (a more precise measure of daily volatility). This evidence supports the hypothesis that these models deliver reasonably accurate volatility predictions. In this paper we concentrate on a broadly used asymmetric GARCH volatility specification (Glosten, Jagannathan and Runkle (1993)) of the form:

$$h_t = \beta_1 + \beta_2 \epsilon_{t-1}^2 + \beta_3 h_{t-1} + \beta_4 1_{(\epsilon_{t-1} \leq 0)} \epsilon_{t-1}^2 \quad , \tag{7}$$

with the indicator function 1_A of the set A taking values 1 if and only if $x \in A$ and zero otherwise. On the other hand, due to the well known difficulties in precisely estimating expected returns (Merton (1980)), we focus on the constant expected returns case, (as in Fleming, Kirby and Ostdiek (2001)):

$$m_t = \alpha_1 \quad . \tag{8}$$

The asymmetric GARCH-M model implied by the the mean equation (8) is estimated by Pseudo Maximum Likelihood (Bollerslev and Wooldridge (1992)). The obtained parameter estimates $\widehat{\vartheta}$ are used as inputs to determine the implied estimated conditionally optimal weights $\widehat{w}_t$ (which are functions only of conditional volatilities). They are given by (cf. also (3)):

$$\widehat{w}_t = \frac{1}{\lambda} \cdot \frac{m_t(\widehat{\vartheta})}{h_t(\widehat{\vartheta})} \quad , \tag{9}$$

II.C Measuring the Effectiveness of Volatility Timing

We analyze the unconditional performance of the estimated dynamic strategies implied by the optimal rules $\widehat{w}_t$ and compare it to that of a fixed weights trading rule using a parametric and a nonparametric resampling methodology: Monte Carlo resampling based on simulations of the estimated parametric mean and volatility processess $\left(m_t^i\left(\widehat{\vartheta}\right)\right)$ and $\left(h_t\left(\widehat{\vartheta}\right)\right)$and block bootstrap (Efron (1979) and Künsch (1989)) based on block resampling of the empirical distribution of the observed data. Each of these resampling techniques is used to generate "artificial" time series of daily excess index returns $(r_t - r_{ft})^{(k)}_{t=0,..,H-1}$, over a desired holding period $[0, H]$. Given an estimated optimal strategy $\widehat{w}_t$ each "artificial" excess index return series $(r_t - r_{ft})^{(k)}_{t=0,..,H-1}$ yields a corresponding "artificial" continuously discounted end of period wealth $W^{(k)}_H$ of the optimal strategy under scrutiny. Repeating this procedure $k = 1, .., N$, times yields a distribution of N "artificial" holding period returns for our estimated volatility strategies.

By parametric simulation, we investigate the effectiveness of volatility timing when the parametric model underlying our volatility strategies is correct. This exercise allows us first to assess if volatility timing has value at all, given a correct model structure, when estimation risk is present.

By nonparametric block bootstrap we analyze if volatility has value when the data are blockwise randomly generated by a refined empirical support of the data (rather than by one of the parametric models used to contruct our dynamic strategies). Hence, nonparametric bootstrap allows us to investigate the impact of some forms of refined "historical" model risk on the performance of our trading rules.

For each relevant investment horizon we provide a marginal and a global description of the "artificial" unconditional performance of volatility strategies. First, we estimate standard unconditional Sharpe Ratios and some VaR and Expected Shortfall modified versions of an unconditional Sharpe Ratio (we denote these ratios by SR, $VaRR^{\alpha}$ and ESR^{α}, respectively, see below). Second, we draw the unconditional mean-variance, mean-VaR and mean-Expected Shortfall frontiers for both strategies under scrutiny. The impact of the investment horizon is analyzed by comparing how these entities vary over investment horizons H between 1 and 12 monthes.

While VaR is by far the most popular and broadly used measure of downside risk in the praxis it is well known that it possesses some conceptual drawbacks, especially when used in an intertemporal portfolio context with several risky assets. Specifically, VaR has difficulties in aggregating individual risks and it may discourage diversification (cf. again Artzner et al. (1998)). Second, in a multiperiod setting a VaR-constrained investor frequently chooses a larger risk exposure than an otherwise equivalent unconstrained investor[14] (as shown for example in Basak and Shapiro (1998)). These two paradoxical features are not inherited by Expected Shortfall. Therefore, we look at both these risk measures to assess the effectivenes of volatility timing in dynamic portfolio management.

The Sharpe Ratio related performance measures used in the paper are defined by:

$$SR = \frac{E_0\left(\ln W_H\right)}{\sqrt{Var_0\left(\ln W_H\right)}}, \quad VaRR^{\alpha} = \frac{\ln E_0\left(W_H\right)}{VaR^{\alpha}\left(W_H\right)}, \quad ESR^{\alpha} = \frac{\ln E_0\left(W_H\right)}{ES^{\alpha}\left(W_H\right)}, \tag{10}$$

where the unconditional VaR and Expected Shortfall measures $VaR^{\alpha}\left(W_H\right)$ and $ES^{\alpha}\left(W_H\right)$ are defined by:

$$\alpha = P_0\left(W_H - 1 + VaR^{\alpha}\left(W_H\right) < 0\right) \quad ,$$

$$ES^{\alpha}(W_H) = -E_0\left((W_H - 1)\,|W_H - 1 + VaR^{\alpha}(W_H) < 0\right) \quad .$$

Notice that $E_0(\cdot)$, $Var_0(\cdot)$ and $P_0(\cdot)$ are the unconditional expectations, variances and probabilities associated to the Monte Carlo and bootstrap distributions of W_H in each of our resampling experiments. Intuitively, $VaR^{\alpha}(W_H)$ is the amount of reserve capital that is needed ad the end of the investment period in order to maintain the probability of a risky loss under a prescribed level α. Similarly, $ES^{\alpha}(W_H)$ is the excess expected loss when considering only excess risky losses exceeding $VaR^{\alpha}(W_H)$ in absolute value, that is losses that cannot be covered by the reserve amount implied by $VaR^{\alpha}(W_H)$.

We estimate the Sharpe Ratio-related performance measures (10) by their empirical conterparts:

$$\begin{aligned}
\widehat{SR} &= \frac{\widehat{E}_0(\ln W_H)}{\sqrt{\widehat{Var}_0(\ln W_H)}},\ \widehat{VaRR}^{\alpha} = \frac{\ln\left(\widehat{E}_0(W_H)\right)}{\widehat{VaR}^{\alpha}(W_H)}, \\
\widehat{ESR}^{\alpha} &= \frac{\ln\left(\widehat{E}_0(W_H)\right)}{\widehat{ES}^{\alpha}(W_H)},
\end{aligned} \tag{11}$$

where:

$$\begin{aligned}
\widehat{E}_0(\ln W_H) &= \frac{1}{N}\sum_{k=1}^{N} \ln W_H^{(k)} \quad , \\
\widehat{Var}_0(\ln W_H) &= \frac{1}{N}\sum_{k=1}^{N} \left(\ln W_H^{(k)} - E_0\left(\widehat{\ln W_H}\right)\right)^2 \quad , \\
\widehat{E_0}(W_H) &= \frac{1}{N}\sum_{k=1}^{N} W_H^{(k)} \quad ,
\end{aligned}$$

and $\widehat{VaR}^{\alpha}(W_H)$, $\widehat{ES}^{\alpha}(W_H)$ are defined by:

$$\begin{aligned}
\alpha &= \frac{1}{N}\sum_{k=1}^{N} 1_{\left(W_H^{(k)} - 1 + \widehat{VaR}^{\alpha}(W_H) < 0\right)} \quad , \\
\widehat{ES}^{\alpha}(W_H) &= -\frac{1}{\alpha N}\sum_{k=1}^{N} \left(W_H^{(k)} - 1\right) 1_{\left(W_H^{(k)} - 1 + \widehat{VaR}^{\alpha}(W_H) < 0\right)} \quad .
\end{aligned}$$

III Results

In this section we present the results of our empirical analysis. After describing our data and discussing some preliminary summary statistics and M-GARCH parameter estimates we first investigate by simulation how estimation risk affects the effectiveness of volatility strategies, assuming constant expected returns. After a standard unconditional mean-variance analysis we extend the discussion to mean VaR/Expected Shortfall considerations. Here, we also present an empirical illustration of the well-known drawbacks of a pure mean-variance analysis when non-normality of returns has to be taken into account. In a second subsection we perform analogous investigations by taking into account model risk through resampling procedures based on nonparametric block bootstrap.

III.A Data and Preliminary Analysis

We consider daily closing prices from June 1989 to January 1999 for four main stock indices: the S&P500, the Dow Jones Industrials, the FTSE 100 and the Nikkei. The source for all indices is Datastream International. Table 1 presents summary statistics of our return series.

Insert Table 1 here

By means of daily mean statistics the S&P500 shows the highest return, followed by the DOW and the FTSE. Remark the negative mean performance of the Nikkei, due to the weakness of the Japanese market over our sampling period. In terms of unconditional estimated standard deviations the S&P500, the FTSE and the Dow have behaved similarly. The Nikkei shows a standard deviation that is almost twice that of the further indices. Finally, higher moments statistics evidence strong nonnormalities.

Asymmetric M-GARCH(1,1) Quasi Maximum Likelihood parameter estimates of model (7), (8) are presented in Table 2. Bollerslev-Wooldridge (1992) robust t-statistics are given in parentheses.

Insert Table 2 here

The estimated mean parameters α_1 are rather consistent with the summary mean statistics discussed above. The FTSE and the S&P500 indices yield the most persistent volatilities, followed by the DOW and the Nikkei. On the other hand, all indices show strongly significant asymmetries in the volatility process, with particularly high estimated parameters for the Nikkei. The response to symmetric shocks is significant and similar across all indices. Finally, notice that the parameter estimates for the FTSE imply an almost integrated volatility process[15].

III.B Quantifying the Effectiveness of Volatility Timing

We consider the distribution of discounted "artificial" end-of-period portfolio values $W_H^{(k)}$ implied by the estimated volatility strategy (9) for holding periods horizons of H=20, 60, 120 and 250 days, respectively.

The next sections investigate the extent to which these strategies enhance the *unconditional* trade-off between return and risk relatively to standard deviation and (or) VaR/Expected Shortfall.

III.B.1 The Impact of Estimation Risk

We first focus on Monte Carlo simulations of the estimated volatility process (7) and generate 10000 artificial daily time series of excess index returns over the given holding period of length H. 10000 time series of portfolios excess returns are then constructed for the optimal estimated strategies (9), yielding 10000 "artificial" end of period discounted wealths $W_H^{(k)}$ for each strategy and investment horizon under scrutiny.

Unconditional Mean Variance Analysis. Table 3 presents estimated annualized Sharpe Ratios SR (cf. (10)) and 95% confidence bounds for a straightforward fix weight strategy and the mean variance strategy (3). The last row of the table provides asymptotic t-tests for the null-hypothesis of a zero difference in the Sharpe Ratios of the two strategies[16]. The investment horizon in this table has been fixed to $H = 60$ days. The results for horizons H = 20, 120, 250 are similar and are omitted.

Insert Table 3 here

Estimated Sharpe Ratios range between 65%-71% for the S&P500, 56%-60% for the DOW, 36%-39% for the FTSE and 9%-19% for the Nikkei, with differences that are significant in favour of the dynamic rule for all indices analyzed. The mean variance frontiers implied by the two strategies are given in Figure 1 for the DOW (for all horizons under investigation)[17], and in Figure 2 for the S&P500 (only for[18] an horizon H=60). The frontiers obtained for the other indices are similar and are omitted[19].

Insert Figure 1-2 here

In all these graphs, the differences observed between fix weight and dynamic strategies for given level of risk or expected return are small. For instance, when $H = 60$ in the case of the Dow (cf. top left panel in Figure 1) and the S&P500 (cf. Figure 2) an investor with risk aversion[20] $\lambda = 5$ loses an ex post estimated difference in annualized mean excess return of no more than 0.5%-1%, when switching from a dynamic to a static strategy and for the same level of standard deviation. On the other hand, for given level of annualized return the differences in standard deviation units for this same investor are[21] about 0.5%-1.5%.

As a consequence, no consistent economically relevant evidence of an enhanced risk-return trade-off between dynamic strategies and fix weight rules is observed by means of standard mean variance criteria in our simulations.

Looking Behind Mean Variance. The mean variance model of asset prices has been analyzed extensively in finance since its development by Markowitz (1952) and is broadly used by practitioners. If fact, a preference for expected return and an aversion to variance can be motivated by monotonicity and strict concavity of individual's utility function within a standard expected utility paradigm[22]. However, for arbitrary return distributions expected utilities cannot be defined only in terms or expected returns and variances. Clearly, this is a particularly relevant point for non-normal and generally for leptokurtic and (or) asymmmetric return distributions. Notice that the existence of GARCH volatilities already implies a leptokurtotic unconditional distribution even for conditionally normal returns (cf. MilhØj (1985) and

Bollerslev (1986)). Similarly, asymmetric GARCH volatilities generate unconditional skewness even when returns are conditionally symmetric. Therefore, it appears quite natural to look behind mean-variance when analyzing the effectivenes of volatility strategies in the presence of (asymmetric) GARCH effects.

As an illustration Figure 3 presents an histogram of the end of period wealth values $W_H^{(k)}$ simulated over an investment horizons of $H = 60$ days based on the estimated asymmetric M-GARCH parameters for the DOW (cf. again Table 2).

Insert Figure 3 here

The lighter histogram represents the distribution of end of period wealth obtained for a fixed weight stategy. The darker histogram represents a corresponding simulated distribution for the conditionally optimal mean variance strategy (3) with estimated weights $\widehat{w}_t$ given by (9). Clearly, the lighter histogram is skewed to the left while the darker one is to the right. Hence, non-normality of holding period returns matters even at horizons of about 3 months in this context[23]. Looking at the corresponding cumulative distributions in Figure 5 an even more interesting feature emerges.

Insert Figure 4 here

Indeed, the cumulative distribution function of end of period wealth for the dynamic strategy (given by the dashed line in Figure 4) stochastically dominates that of the fixed weights strategy (the second line in Figure 4). Similar features are found for the S&P500 and partly for the FTSE500.

Clearly, this feature would be missed by a pure mean-variance analysis. The motivation for evaluating volatility strategies also by other risk measures than variance in order to avoid counter-intuitive risk assessments is confirmed by this simple empirical example.

Unconditional Mean Var/Expected Shortfall Analysis. Table 4 and 5 present the estimated "downside risk" modified Sharpe Ratios $VaRR^\alpha$ and ESR^α in (10) (with 95% confidence bounds) estimated in our simulations. As for the standard Sharpe Ratios the investment horizon in these tables is $H = 60$ days. The results for horizons $H =$ 20, 120, 250 are similar and are omitted.

Insert Table 4-5 here

Estimated VaR-modified (Expected Shortfall-modified) Sharpe Ratios in Table 4 range between 68%-104% (52%-91%) for the S&P500, 59%-83% (48%-72%) for the DOW, 36%-47% (29%-41%) for the FTSE and 11%-17% (7%-15%) for the Nikkei. Compared to the standard Sharpe Ratio case in Table 3 differences in performance are now more significant for all indices with exception of the Nikkei (cf. the t-statistics in the last row of Table 4 and 5).

These differences seem to be also economically more relevant than those obtained by a pure mean variance analysis. For instance, in the unconditional frontiers in Figure 5 an investor with risk aversion $\lambda = 5$ following a volatility timing strategy in the DOW at horizons $H = 60$ gets a higher annualized excess returns of about 3.5%, compared to a fix weight investment having the same 1%-VaR. This same difference amounts to more than 4% for the S&P500 (cf. Figure 6 below). For the FTSE the gain in mean return is less significant and for the Nikkei it is factly negligigle.

Insert Figure 5-6 here

On the other hand, the gain in VaR and Expected Shortfall achieved by the same investor at a given targeted mean return appears to be even more relevant. For instance, in the case of the DOW and the S&P500 one gets a 1%-VaR reduction from about 0.89 to 0.84 and 0.87 to 0.80, respectively, when switching from the dynamic to the static strategy at a given targeted level of expected return. These differences appear to be economically relevant also for the FTSE and the Nikkei where VaR shrinks from 0.93 to 0.90 and from 0.93 to 0.89, respectively. Differences in terms of Mean-Expected Shortfall frontiers are even more clear-cut (cf. for instance Figure 7 for the S&P500)[24].

Summarizing, our simulations suggest that for the DOW and the S&P500 volatility timing and related strategies ameliorate the risk-return index investment profile both by enhancing return at a given level of downside risk and by lowering downside risk at a given targeted expected return. For the FTSE and the Nikkei only the second of these two effects appears to be economically significant.

III.B.2 The Impact of Model Risk

We make use of nonparametric bootstrap to generate 10000 artificial daily time series of excess index returns over a given holding period of length H. 10000 time series of portfolios excess returns are then constructed for the optimal strategy (9), yielding 10000 "artificial" end of period wealth level $W_H^{(k)}$ for each strategy and investment period under scrutiny.

Block Bootstrap. Block bootstrap is a resampling methodology that allows for a stationary dependence structure in the underlying process (at variance with the original i.i.d. bootstrap by Efron (1979)). This is an important feature when investigating stochastic processes where GARCH volatilities can induce quite strong dependence features. At variance with Fleming, Kirby and Ostdiek (2001) we therefore make use of a particular block bootstrap methodology with overlapping blocks (Künsch (1989)) in order to produce a refined nonparametric estimator of the historical data distribution. This allows us to assess how far a misspecification of our GARCH models could affect the performance of the strategies under scrutiny.

We bootstrap directly the index returns empirical distribution, rather than - as done by others (cf. for instance Barone Adesi, Giannopoulos and Vosper (1999)) - using an i.i.d bootstrap procedure on the filtered GARCH residuals. This fully nonparametric approach is not dependent on the supposed parametric GARCH volatility structure. However, assessing the validity of the hypotheses used and interpreting the empirical evidence in a full nonparametric setting can be sometimes cumbersone (see for instance Maddala (1996) for a review). Therefore, we check carefully for consistency of our bootstrap results with the above parametric simulation evidence.

Unconditional Mean Variance Analysis. Table 6 presents estimated annualized Sharpe Ratios SR (cf. (10)) with 95% confidence bounds, estimated in our bootstrap experiments for a fix weight strategy and the optimal mean variance strategy (3), respectively. The investment horizon in this table has been fixed to $H = 60$ days. The results for horizons $H = 20$, 120, 250 are similar and are omitted.

Insert Table 6 here

Estimated Sharpe Ratios range between 89%-67% for the S&P500, 69%-60% for the DOW, 26%-22% for the FTSE and 24%-34% for the Nikkei. Somehow surprisingly, the mean-variance rule (3) performs worse for all indices (with strongly significant t-tests) with the exception of the Nikkei.

While this seems to suggest a poor effectiveness of volatility strategies in our bootstrap experiments, the next section shows that this finding is completely determined by the inadequacy of mean variance criteria when used to rank non-normal return pay-offs.

Unconditional Mean Var/Expected Shortfall Analysis. Table 7 and 8 present the estimated "downside risk" modified Sharpe Ratios $VaRR^{\alpha}$ and ESR^{α} in (10) (with 95% confidence bounds), estimated in our bootstrap experiments. Again, the investment horizon in these tables is $H = 60$ days. The results for horizons $H = 20$, 120, 250 are similar and are omitted.

Insert Table 7-8 here

Estimated VaR-modified (Expected Shortfall-modified) Sharpe Ratios $VaRR^{\alpha}$ (ESR^{α}) range between 100%-106% (83%-93%) for the S&P500, 76%-86% (64%-77%) for the DOW, 29%-28% (25%-25%) for the FTSE and 24%-29% (20%-24%) for the Nikkei. Differences between static and dynamic strategies are statistically significant in favour of dynamic strategies, with exception of the FTSE presenting no significant test statistic.

Only for the Nikkei, similar strategy rankings to those obtained by standard Sharpe Ratios in the last section arise. On the other hand, for the S&P500, the DOW and partly for the FTSE different strategy orders arise. In fact, by a standard mean variance analysis we have seen that fix weights strategies would be tendentially preferred to dynamic ones for these three indices. However, as for the distribution obtained by parametric simulation we observe in Figure 8 an end of period wealth bootstrap distribution for the fix weight strategy that is stochastically dominated by that of the dynamic strategy (9) in the case of the DOW. The same feature is found for the S&P500 end of period wealth bootstrap distribution.

Insert Figure 8 here

Again a pure mean-variance criterium does not tell the whole story when comparing short-term volatility strategies. It appears, that a further look at downside risk tends to avoid a paradoxical conclusion on the effectiveness of volatility strategies.

The differences obtained with respect to a downside risk criterium seem to be economically relevant also in the presence of model risk.

Insert Figure 9 here

For example, in the unconditional frontiers in Figure 9 an investor with $\lambda = 5$ obtains in the case of the Dow and at horizons $H = 60$ an expected excess return that is about 1.5% higher that that obtained by a fix weights rule having the same unconditional 1%-VaR[25]. On the other hand, for given targeted expected excess return switching to the "static" rule implies a VaR falling from[26] about 89% to about 87%. In the case of the Nikkei, this same investor reduces expected excess returns by about 0.5%-1% for given VaR when investing in the fixed weight rule. Finally, by given expected excess returns the implied VaR reduction in the Nikkei case is between 1%-3%. Differences in terms of Expected Shortfall are even more clear-cut[27].

As a conclusion, the bootstrap evidence suggests that volatility timing and related strategies are effective in ameliorating the downside risk-return investment profile of market portfolio positions. This evidence is consistent with the simulation results obtained above.

IV Conclusions

We have found a substantial effectiveness of volatility strategies in enhancing the trade-off between market downside risk and market return in the presence of both estimation and model risk. These effects are robust to transaction cost amounts as they are typically required for futures positions on liquid markets. On the other hand, mean-variance comparisons do not yield a consistent evidence in favour of volatility strategies because they often paradoxically select pay-offs that are stochastically dominated by those of dynamic volatility strategies. Topics for future research are first the analysis of the relation between volatility timing effectiveness and index structure and second the investigation of the effectiveness of volatility strategies in ameliorating the risk return profile of dynamic asset allocations for private investors.

Notes

[1]In this last framework, it is known at least since Merton (1969) that CRRA investors allocate a constant fraction of current wealth to risky assets.

[2]Moreover, optimal intertemporal allocations do then generally depend on the investor's horizon, thereby making time diversification less effective (see for instance Samuelson (1989), (1990) for a discussion).

[3]References documenting predictability of expected returns using different sets of economic intrumental variables are Campbell (1987), Campbell and Shiller (19988a, 19988b), Cochrane (1991), Fama and Schwert (1977), Fama and French (1988, 1989), Ferson and Harvey (1991), Keim and Stambauch (1986), Lamont (1998), Lettau and Ludvigson (1999), Pontiff and Shall (1998). Some prominent studies on variance predictability are Andersen and Bollerslev (1998), Bollerslev (1986), Bollerslev Engle and Nelson (1994), Diebold and Lopez (1995), Engle (1982), Harvey (1991), Palm (1996), Schwert (1989) and Whitelaw (1994).

[4]As in many papers on expected returns predictability, this evidence is based on parameter estimates obtained using monthly data.

[5]While this suggests that many fund managers behave like volatility timers their trading decisions may however be driven by other factors than volatility.

[6]The economic value of volatility timing is quantified by an estimated annual fee of no less than about 100 basis points (when estimation risk is considered) for switching from the static to the dynamic strategy.

[7]See for instance Ané and Geman (2000) for a nice economic interpretation of this point.

[8]As a consequence, at daily frequencies the problem of synthesizing returns predictability by an appropriate index of predictive variables (see for instance Ait-Sahalia and Brandt (2000)) for dynamic portfolio management purposes seems to be less stringent than at monthly frequencies.

[9]As has ben shown in Whitelaw (2000) these empirical facts are not necessarily inconsistent with financial intuition. In that paper, a non linear and time-varying relation duplicating the salient feature of the risk/return trade-off in the data is estimated based on a switching regime general equilibrium model with state dependent regime probabilities.

[10]The same solution is obtained for an investor maximizing exponential conditional expected utility.

[11]The multiperiod unconditional mean variance problem has been solved recently by Li and Ng (2000), for the case of i.i.d. asset returns.

To our knowledge, no solution for the stochastic volatility case has been derived yet in the literature.

[12]Alternatively, the optimal weights (3) could be estimated directly by a nonparametric estimator using a conditional Euler equation approach (see Brandt (1999) and Ait Sahalia and Brandt (2000)). Since we are interested in assessing the value of well-known *parametric* volatility forecast models we do not follow such a nonparametric approach here. Clearly, a non parametric estimation of the optimal weights could produce insights about a misspecification of the optimal weights implied by a parametric model. On the other hand, if the given parametric model is misspecified the estimated portfolio rules are systematically sub-optimal. Hence, if volatility timing has value in reducing downside risk when using a suboptimal strategy it is likely that more involved strategies will yield even better performances.

[13]Some rewiews of this research are provided by Bollerslev, Chou and Kroner (1992), Bollerslev, Engle and Nelson (1994) and Diebold and Lopez (1995).

[14]This second feature could be more relevant for our analysis since we focus on simple market timing strategies rather than on dynamic portfolios of several risky assets.

[15]For this reason the parametric and nonparametric simulation experiments for the FTSE in the next sections have to be interpreted with caution.

[16]This test makes use of the asymptotic distributions of the statistics, which are available from the authors.

[17]The underlying risk aversion parameters λ ranges between 2.5 and 10. Circles correspond to risk aversions λ=2.5, 5, 10, respectively.

[18]Since, as for the DOW (cf. Figure 1) no numerically important horizon effects in mean standard deviation space are observed.

[19]The corresponding figures are available from the authors.

[20]For this risk aversion level the mean position in the index of a dynamic volatility strategy is approximately 1.

[21]For the other indices the differences obtained are smaller.

[22]This is immediately seen by a second order Taylor expansion of the individual's utility function.

[23]As a comparison, remember that under normality of holding period returns end of period wealth is lognormaly distributed.

[24]Further graphs for the Expected Shortfall case are available from the authors.

[25]For the S&P500 this same increase in expected return amount to

about 1%.

[26]For the S&P 500 index 1%-VaR decreases from about 88% to about 86%.

[27]Details are available from the authors.

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TABLES

TABLE 1

	S&P500	**DOW**	**FTSE**	**NIKKEI**
Mean	0.000471	0.000381	0.000164	-0.000313
Median	0.000237	0.000171	-0.000100	-0.000103
Maximum	0.049793	0.048451	0.054103	0.124083
Minimum	-0.071221	-0.074704	-0.041658	-0.068467
Std. Dev.	0.008791	0.008856	0.008978	0.014552
Skewness	-0.465679	-0.572547	0.054215	0.330777
Kurtosis	9.174395	9.633212	5.225211	7.419384
Jarque-Bera	4388.065 (0.00000)	5099.347 (0.00000)	558.5802 (0.00000)	2247.301 (0.00000)

TABLE 2

	S&P500	**DOW**	**FTSE**	**NIKKEI**
$\boldsymbol{\alpha}$	0.000398 (2.920641)	0.000338 (2.313962)	0.000200 (1.321809)	-0.000207 (-0.997177)
β_1	$8.78E-07$ (3.565701)	$1.82E-06$ (3.288707)	$5.15E-07$ (2.533461)	$2.46E-06$ (3.242123)
β_2	0.012544 (1.201800)	0.015005 (1.371729)	0.014154 (2.105027)	0.018459 (1.668012)
β_3	0.943024 (87.76735)	0.926882 (65.45508)	0.960000 (130.2840)	0.911703 (76.43995)
β_4	0.066477 (3.942419)	0.067758 (3.096684)	0.036631 (3.737376)	0.125399 (5.173562)

TABLE 3

Simulation	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
SR				
static	0.649 ± 0.046	0.558 ± 0.044	0.356 ± 0.042	0.087 ± 0.040
dynamic	0.710 ± 0.038	0.602 ± 0.039	0.387 ± 0.039	0.195 ± 0.041
$t - stat$	7.905	10.625	4.544	6.692

TABLE 4

Simulation	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
$\frac{\ln \mathbf{E}(\mathbf{V}_H)}{VaR(V_H,H,\alpha)}$				
static	0.675 ± 0.042	0.587 ± 0.048	0.363 ± 0.042	0.109 ± 0.041
dynamic	1.045 ± 0.062	0.826 ± 0.057	0.466 ± 0.048	0.172 ± 0.033
$t - stat$	18.449	14.164	7.556	3.888

TABLE 5

Simulation	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
$\frac{\ln \mathbf{E}(\mathbf{V}_H)}{ES(V_H,H,\alpha)}$				
static	0.519	0.476	0.290	0.074
dynamic	0.911	0.721	0.407	0.147

TABLE 6

Bootstrap	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
SR				
static	0.891 ± 0.043	0.691 ± 0.043	0.262 ± 0.040	0.236 ± 0.040
dynamic	0.673 ± 0.037	0.598 ± 0.038	0.224 ± 0.039	0.345 ± 0.043
$t - stat$	-23.085	-13.178	-5.457	8.796

TABLE 7

Bootstrap	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
$\frac{\ln \mathbf{E}(\mathbf{V}_H)}{VaR(V_H,H,\alpha)}$				
static	1.000 ± 0.046	0.759 ± 0.039	0.294 ± 0.020	0.237 ± 0.006
dynamic	1.059 ± 0.031	0.858 ± 0.024	0.281 ± 0.010	0.287 ± 0.010
$t - stat$	2.771	4.920	-1.289	3.345

TABLE 8

Bootstrap	**S&P500**	**DOW**	**FTSE**	**NIKKEI**
$\frac{\ln \mathbf{E}(\mathbf{V}_H)}{ES(V_H,H,\alpha)}$				
static	0.828	0.644	0.252	0.204
dynamic	0.926	0.767	0.248	0.242

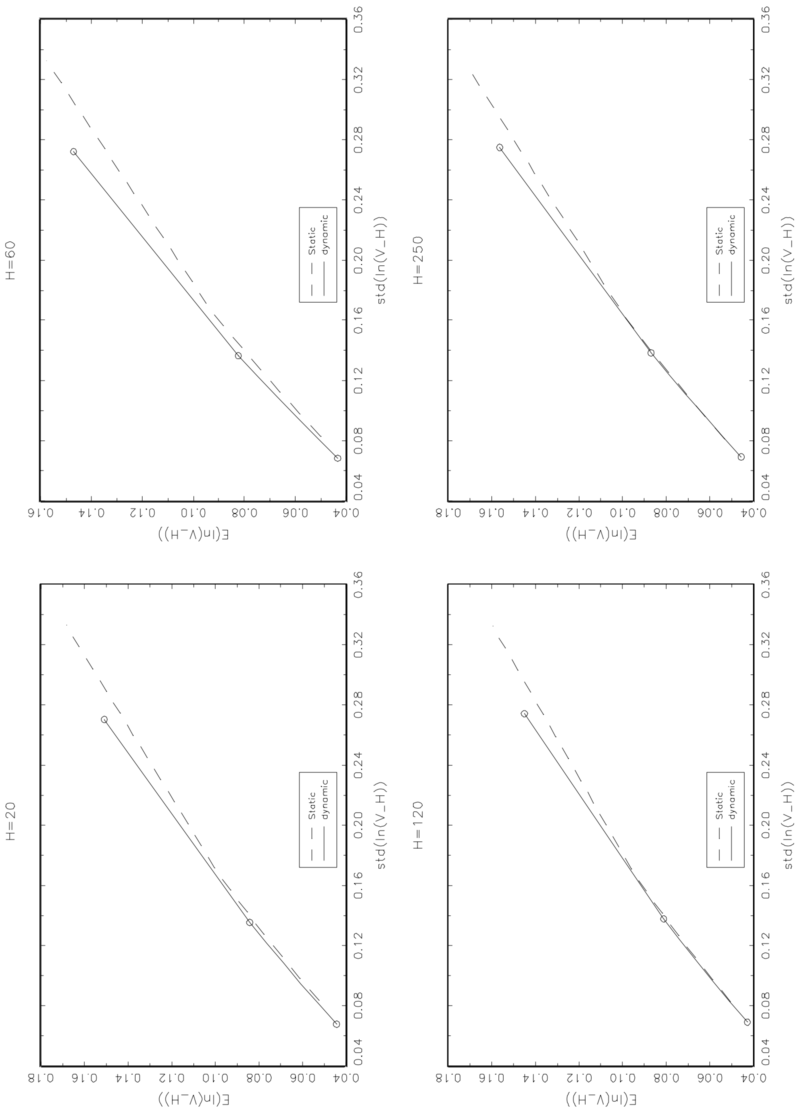

Figure 1: Mean-Variance Analysis using simulations for the DOW at various horizons $H = 20$, 60, 120 and 250.

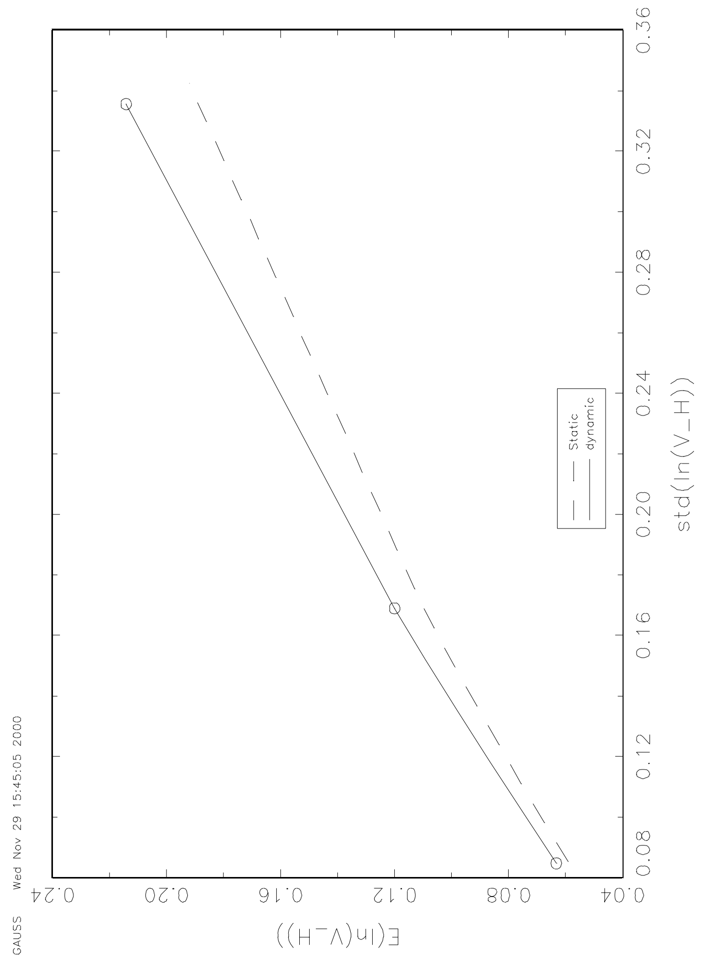

Figure 2: Mean-Variance analysis using simulations for the S&P at horizon $H = 60$.

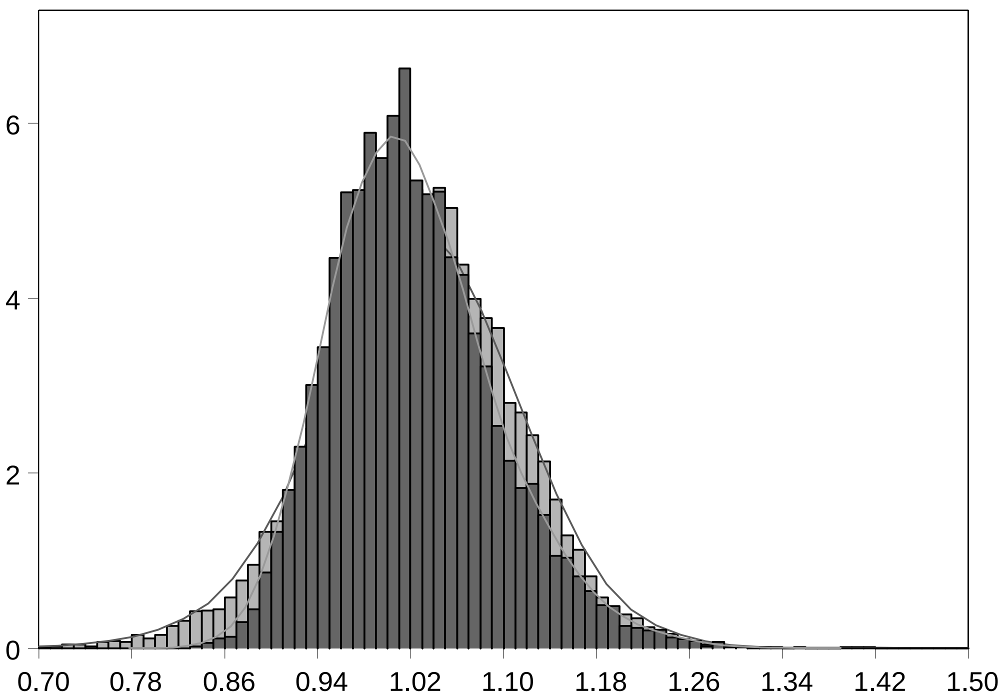

Figure 3: Distributions of simulated end-of-period portfolio value W_H for DOW. Darker distribution is dynamic strategy, lighter distribution is fixed weight strategy.

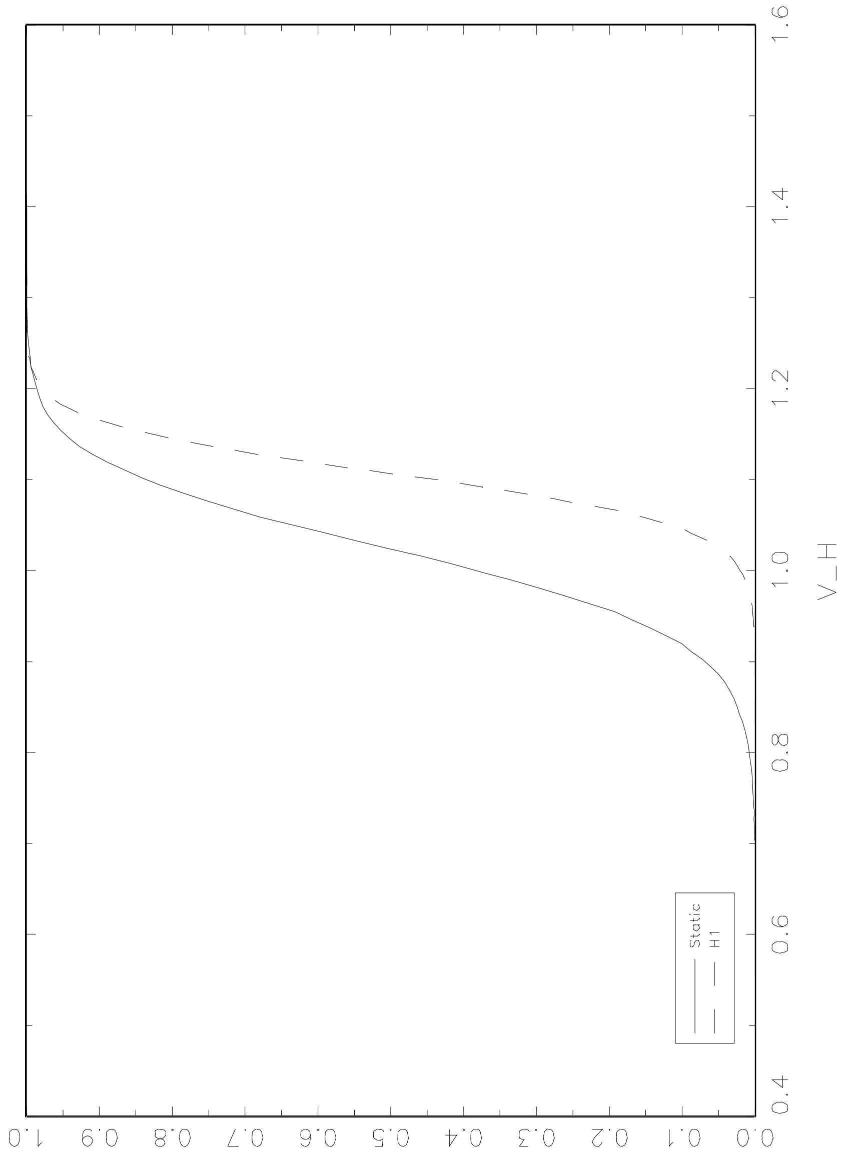

Figure 4: Cumulative distribution functions for V_H in simulation for the DOW.

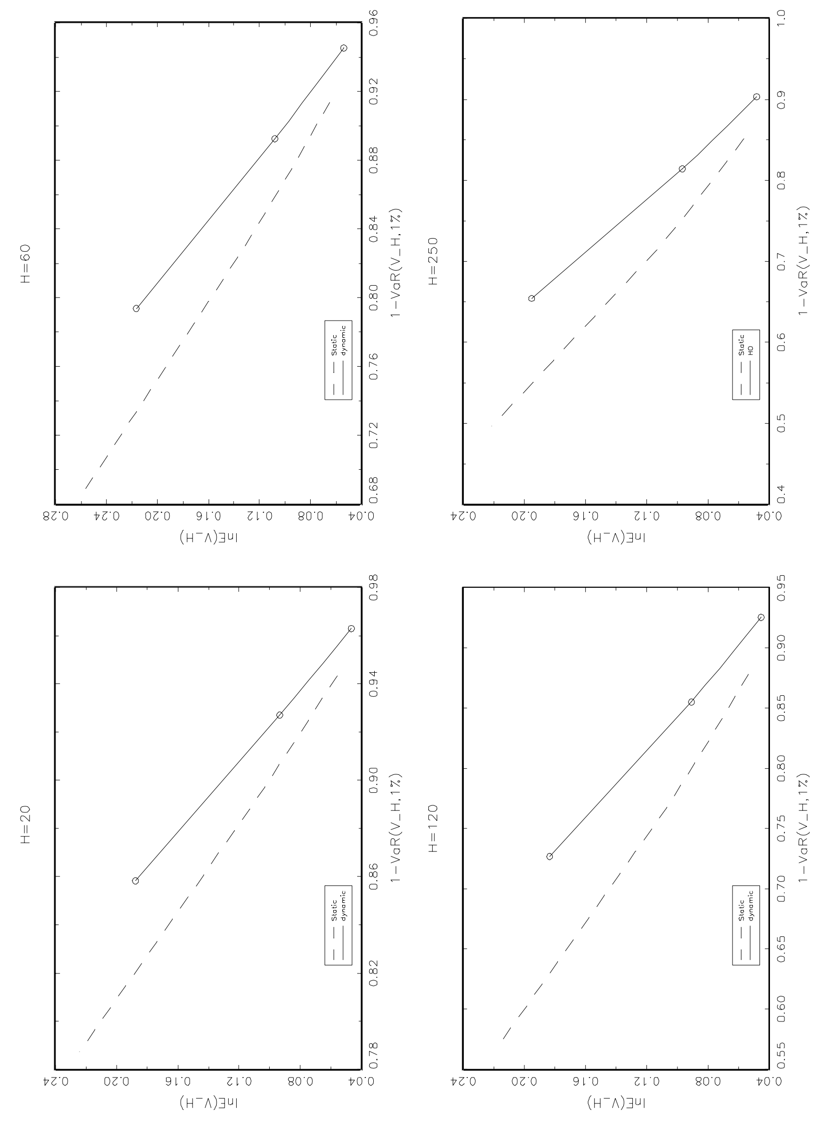

Figure 5: Mean-Value at Risk simulations for DOW at different horizons H = 20, 60, 120 and 250.

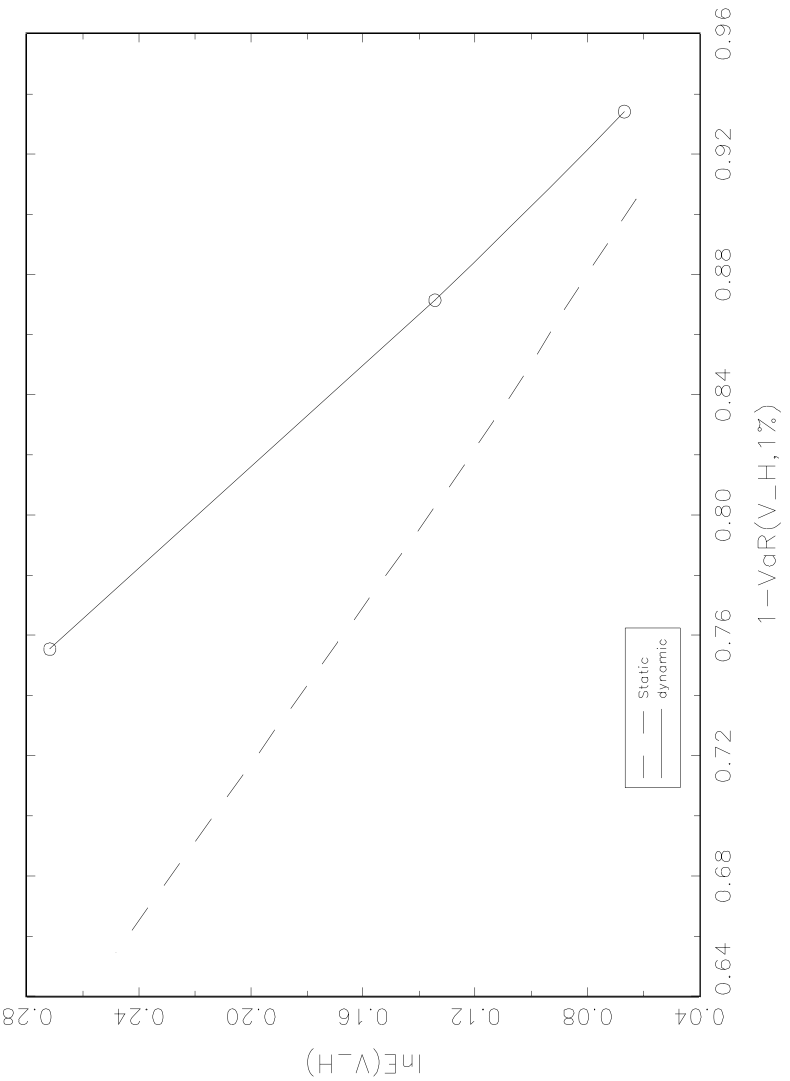

Figure 6: Mean-Value at Risk simulations for S&P.

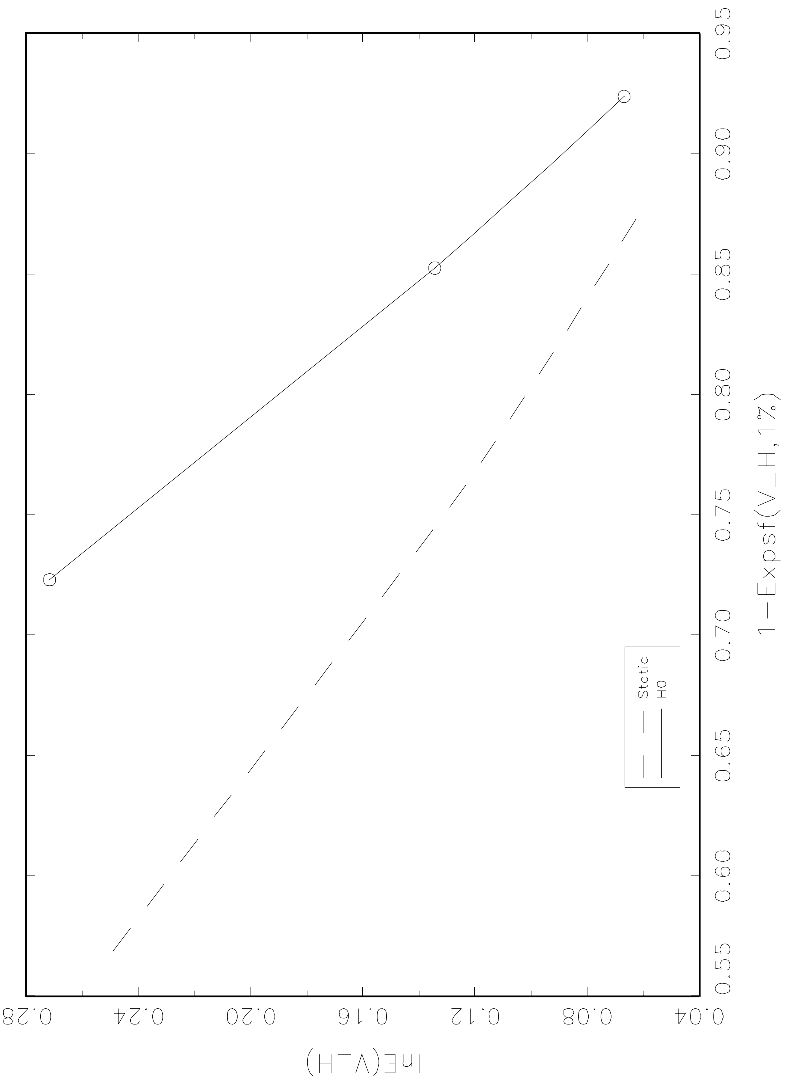

Figure 7: Mean-Expected Shortfall simulations for S&P.

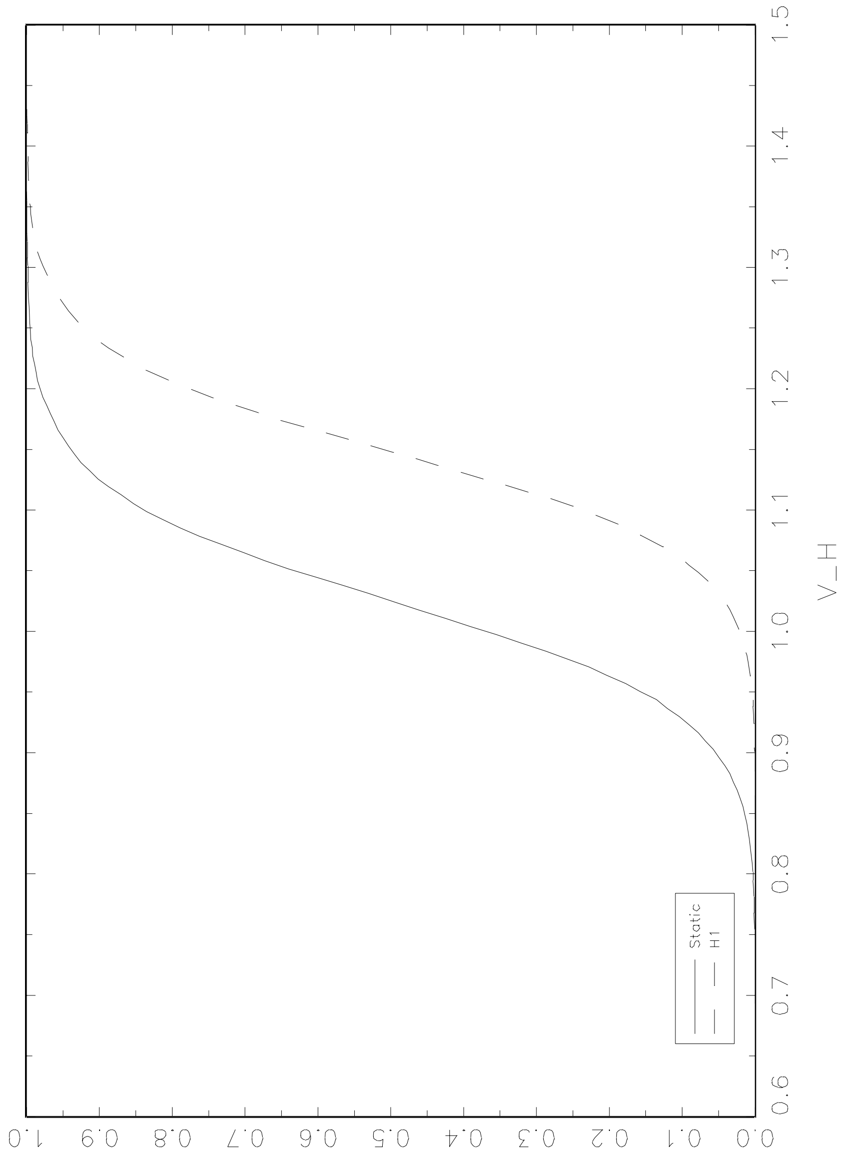

Figure 8: Cumulative distribution function of V_H in bootstrap for the DOW.

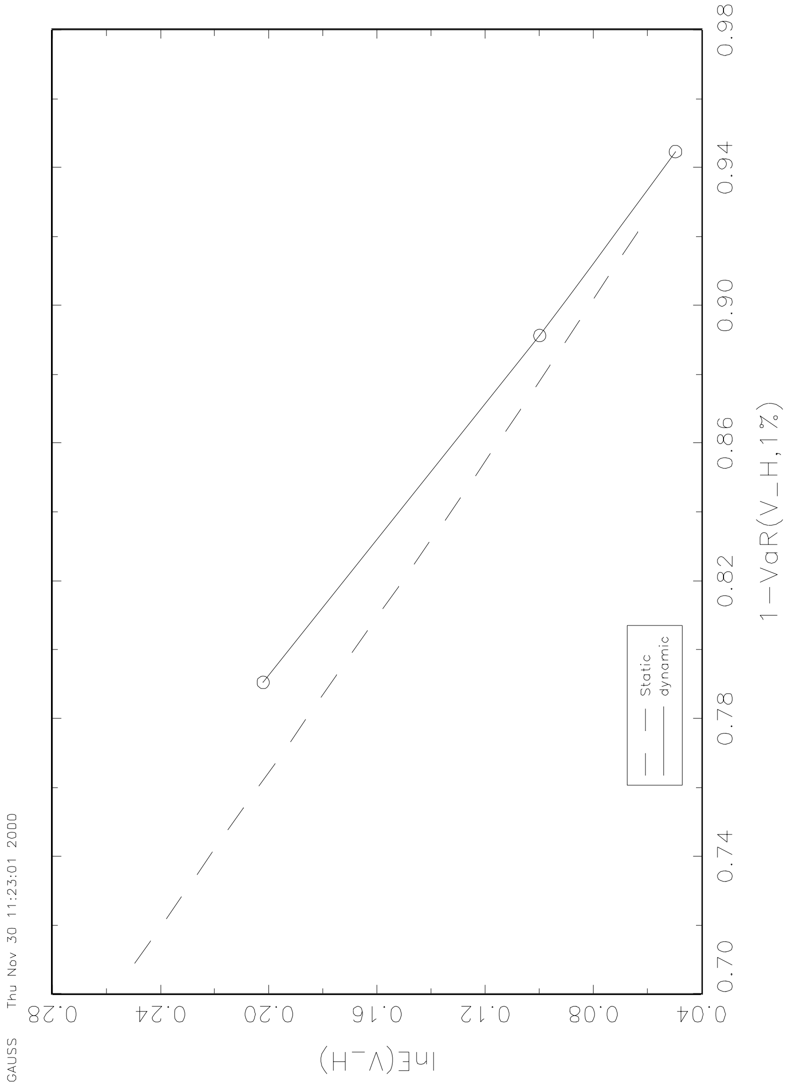

Figure 9: Mean-Value at Risk analysis using bootstrap for DOW at horizon $H = 60$.