

Generalization of the Luttinger Theorem for Fermionic Ladder Systems

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We apply a generalized version of the Lieb-Schultz-Mattis Theorem to fermionic ladder systems to show the existence of a low-lying excited state (except for some special fillings). This can be regarded as a non-perturbative proof for the conservation under interaction of the sum of the Fermi wave vectors of the individual channels, corresponding to a generalized version of the Luttinger Theorem to fermionic ladder systems. We conclude by noticing that the Lieb-Schultz-Mattis Theorem is not applicable in this form to show the existence of low-lying excitations in the limit that the number of legs goes to infinity, e.g. in the limit of a 2D plane.

In their paper of 1961 [1] Lieb, Schultz and Mattis (LSM) showed as an exact result that an antiferromagnetic Heisenberg spin- $\frac{1}{2}$ chain of length L (even) with periodic boundary conditions has gapless excitations in the thermodynamic limit. In one dimension, $S = \frac{1}{2}$ spins can be mapped into spinless fermions by the Jordan-Wigner transformation. As a result, the LSM Theorem should also be applicable to one-dimensional fermion problems. This was demonstrated recently by Yamanaka, Oshikawa and Affleck [2], who considered a translationally invariant Hamiltonian with short-range hopping, which conserves the number of up- and down-spin separately and is invariant under parity or time reversal. Under these assumptions, the generalized LSM Theorem can be stated as follows: in a chain of length L with periodic boundary conditions an excited state with energy $O(\frac{1}{L})$ above the ground state exists at momentum $2\pi\nu_\sigma$, if the density ν_σ of fermions with spin σ per unit cell is not an integer and the ground state is not degenerate.

In a non-interacting one-dimensional system, the presence of gapless excitations at momentum $2k_F$ is a consequence of the existence of Fermi points at $\pm k_F$. Luttinger showed perturbatively the existence of a meaningful concept of Fermi surface in an interacting system, identifying it as the surface located at the singular wave vectors of the momentum distribution function $n(\mathbf{k})$, which enclose the same volume as in the non-interacting system [3]. Luttinger's proof applies to systems belonging to the Landau-Fermi liquid universality class, and hence not directly to 1D problems. The existence of gapless excitations at $2k_F = 2\pi\nu_\sigma$ in an interacting system can be seen as a generalized version of the Luttinger Theorem in one dimension.

The application of the LSM Theorem to spinful electrons interacting on a ladder allows us to obtain a generalized Luttinger Theorem for such systems. In this geometry, we consider the Hamiltonian:

$$H = -t \sum_{\alpha=1}^{\lambda} \sum_{j,\sigma} c_{\alpha,j+1,\sigma}^\dagger c_{\alpha,j,\sigma} + H.c.$$

$$-t' \sum_{\alpha=1}^{\lambda-1} \sum_{j,\sigma} c_{\alpha+1,j,\sigma}^\dagger c_{\alpha,j,\sigma} + H.c. + H_{int} = H_t + H_{int}$$

where $c_{\alpha,j,\sigma}$ is the annihilation operator for a fermion on the chain α , rung j , and with spin σ . t and t' are the hopping amplitudes along the chains and along the rungs respectively. Periodic boundary conditions are used along each chain: $c_{\alpha,j+L,\sigma} = c_{\alpha,j,\sigma}$, $\alpha = 1, \dots, \lambda$ (see Fig 1).

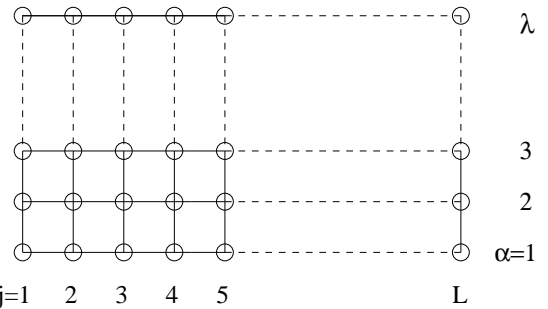


FIG. 1. A λ -leg ladder. The lattice points are denoted by small circles.

We assume that H_{int} involves the local fermion densities only, that H preserves the number of up- and down-spin separately, and that it is translationally and parity invariant. Here translation and parity operators are defined by $Tc_{\alpha,j,\sigma}T^{-1} = c_{\alpha,j+1,\sigma}$, and $Pc_{\alpha,j,\sigma}P^{-1} = c_{\alpha,-j,\sigma} = c_{\alpha,L-j,\sigma}$, respectively.

Defining the twist operator U_σ , $\sigma = \uparrow, \downarrow$, by

$$U_\sigma = \exp(2\pi i \sum_{\alpha,j} \frac{j}{L} n_{\alpha,j,\sigma}), \quad (1)$$

we obtain $U_\sigma^{-1} c_{\alpha,j,\sigma}^\dagger U_\sigma = e^{-i\frac{2\pi}{L}j} c_{\alpha,j,\sigma}^\dagger$, and thus H_{int} and the kinetic term for the hopping along the rungs are invariant under U_σ . Letting $|\phi_0\rangle$ be the ground state of the system, the excitation energy of the twisted state $|\phi_1\rangle = U_\sigma|\phi_0\rangle$ is $E_1 = \langle\phi_0|U_\sigma^{-1} H U_\sigma - H|\phi_0\rangle = -t(e^{-i\frac{2\pi}{L}} - 1) \sum_{\alpha,j} \langle c_{\alpha,j+1,\sigma}^\dagger c_{\alpha,j,\sigma} \rangle + H.c.$. Expanding the exponential and assuming a parity invariant ground state, the term $O(1)$, involving expectation values like $\langle i \sum_j c_{\alpha,j+1,\sigma}^\dagger c_{\alpha,j,\sigma} - c_{\alpha,j,\sigma}^\dagger c_{\alpha,j+1,\sigma} \rangle$, vanishes, and we thus obtain: $E_1 = O(\frac{\lambda}{L})$.

From $TU_\sigma T^{-1} = e^{-2\pi i\lambda\nu_\sigma} U_\sigma$ we deduce that the crystal momentum P of $|\phi_1\rangle$, defined by $T = e^{-iP}$, is $2\pi\lambda\nu_\sigma$, relative to the ground state. This proves that the twisted state $|\phi_1\rangle$ is orthogonal to $|\phi_0\rangle$ if $\lambda\nu_\sigma$ is not an integer. In this case, the existence of at least one low-lying excited state is assured variationally, as long as the ground state is unique.

Thus, the LSM Theorem can be generalized to interacting fermions on a λ -leg ladder as follows: under the above assumptions for the Hamiltonian H and its ground state, gapless excitations exist in the thermodynamic limit at momentum $2\pi\lambda\nu_\sigma$, if $\lambda\nu_\sigma$ is not an integer.

We notice that this method, being essentially topological in nature, does not depend on the details of the Hamiltonian, but only on general properties. For example, if a next-nearest neighbor hopping term is introduced, it gives a contribution of order $O(\frac{\lambda}{L})$ to the excitation energy of the twisted state, and the proof applies as well.

Following Yamanaka et al. [2], we define charge and spin twist operators by

$$U_c = U_\uparrow U_\downarrow, \quad U_s = U_\uparrow U_\downarrow^{-1}. \quad (2)$$

Under their action, the electron spin operators $\mathbf{S}_{\alpha,j} = c_{\alpha,j,\sigma}^\dagger \frac{\tau_{\sigma,\sigma'}}{2} c_{\alpha,j,\sigma'}$ transform like

$$U_c^{-1} \mathbf{S}_{\alpha,j} U_c = \mathbf{S}_{\alpha,j} \quad U_s^{-1} \mathbf{S}_{\alpha,j} U_s = \mathbf{R}_3\left(\frac{4\pi}{L}j\right) \mathbf{S}_{\alpha,j}, \quad (3)$$

where $\mathbf{R}_3(\alpha)$ is the rotation about the third axis with angle α . In this sense, the operators U_c and U_s can be interpreted as twist operators creating a charge and a spin excitation respectively. From $TU_c T^{-1} = U_c e^{-2\pi i\lambda\nu}$, where $\nu = \nu_\uparrow + \nu_\downarrow$ and $TU_s T^{-1} = U_s e^{-2\pi i\lambda(\nu_\uparrow - \nu_\downarrow)}$, we see that gapless charge (spin) excitations exist if $\lambda\nu$ ($\lambda(\nu_\uparrow - \nu_\downarrow)$) is not an integer. If no magnetization is present $\nu_\uparrow = \nu_\downarrow = \frac{\nu}{2}$, a statement about the existence of gapless spin excitations cannot be made by the above method.

At half-filling ($\nu_\sigma = \frac{1}{2}$), a gapless excitation is shown to exist (if the above assumptions hold) in a ladder with an odd number of legs. However, in all cases at this filling a charge gap can open (due to relevant Umklapp processes). Furthermore, unlike the case of a single chain, in a λ -leg ladder a charge gap can open (without breaking the translational symmetry of the ground state) even at rational fillings, i.e. $\nu = \frac{m}{\lambda}$, with m integer. These conclusions are consistent with perturbative and numerical studies [4].

For the Hubbard model, the application of the generalized LSM Theorem leads to conclusions which are independent on the sign of U . However, the physics of the attractive and of the repulsive Hubbard model are very different: on a chain, for $U < 0$, the model scales to strong-coupling and falls in the universality class of the Luther-Emery Liquid [5], with gapped spin excitations, whereas for $U > 0$ it scales to the Tomonaga-Luttinger fixed point [6], with gapless charge and spin

excitations (away from half-filling). (For reviews of interacting fermion systems in one dimension and bosonization techniques, see e.g. [7], [8], [9] and [10]). Hence, the presence of gapless excitations on a Hubbard chain away from half-filling, without characterization of their nature, as obtained with the LSM Theorem using U_σ , is in agreement with perturbative results. However, the differences in the spin excitation spectrum between the various regions of the parametric space cannot be observed with the LSM Theorem, since the momentum of the excitations created by U_s vanishes (when no magnetization is present). To get further insight, we apply the LSM Theorem to the strong coupling limits ($U \rightarrow \pm\infty$) of the Hubbard model separately.

In the attractive case, for large $|U|$, the electrons form Cooper pairs with opposite spins, occupying equally spaced lattice points, and the Hamiltonian reduces (up to constant terms) to one for hard-core bosons on a lattice with a repulsive next-neighbor interaction [11]. An appropriate twist operator can be defined to show that gapless charge excitations exist at momentum $\pi\nu$, if ν is not an even integer and the ground state is not degenerate. This shows that the low-lying excitations at momentum $2\pi\nu_\sigma$, whose existence was shown above at finite U without specifying their nature, are actually charge excitations, in the attractive case. Spin excitations require the breaking of a Cooper pair and cost an energy amount of order $|U|$.

In the case $U > 0$, the strong on-site repulsion favors single occupation of the lattice sites by the electrons, and thus the Hubbard model at strong-coupling scales to the t-J Hamiltonian, for which we can show as well that $E_1 = O(\frac{1}{L})$, noticing that the twist operator acts on the electron spin operator as one would expect: $U_\sigma^{-1} \mathbf{S}_j U_\sigma = \mathbf{R}_3(\pm \frac{2\pi}{L}j) \mathbf{S}_j$, $\sigma = \uparrow, \downarrow$, and that it commutes with the projector $P_s = \prod_j (1 - n_{j\uparrow} n_{j\downarrow})$, which prohibits double occupancy.

Recently, it was proposed that the introduction of a next-nearest-neighbor hopping term in the 1D t-J model could lead to a breakdown of the Luttinger liquid behavior [13]. In particular, numerical calculations have suggested the existence of a low-doping phase with Fermi momentum $k_F = \frac{\pi}{2}\delta = \frac{\pi}{2}(1 - \nu)$ (determined thus by the density of holes), and a high-doping phase with $k_F = \pi\nu$ (determined thus by the density of spinless fermions); neither of these k_F is compatible with a Luttinger liquid. As pointed out above, the generalized LSM Theorem shows the existence of gapless excitations at momentum $2k_F = \pi\nu$ even after the introduction of a next-nearest hopping term, unless a degeneracy in the ground state occurs. The above high-doping phase is not consistent with our $2k_F$ -excitation. The origin of this ‘‘anomalous’’ k_F could lie in ferromagnetic correlations.

The generalization to the case of a λ -leg t-J ladder by means of (1) is immediate: a low-energy excitation at momentum $2\pi\lambda\nu_\sigma$ is explicitly shown to exist in a λ -leg

t-J ladder, if $\lambda\nu_\sigma$ is not an integer. At half-filling, the t-J model reduces to a Heisenberg (spin-only) Hamiltonian, and thus the low-lying excited states for λ odd have the character of spin excitations, as can be shown with the specific twist operator $U = e^{2\pi i \sum_{\alpha,j} \frac{j}{L} \mathbf{s}^3_{\alpha,j}}$, introduced by Affleck [12]. Using the known property of spin- $\frac{1}{2}$ representations $e^{2\pi i S^3_{\alpha,j}} = -1$, and assuming a non-degenerate singlet ground state in the thermodynamic limit, one gets $TUT^{-1} = (-1)^\lambda U$ (acting on the ground state), and thus the relative momentum of the twisted state is π (0) if λ is odd (even).

We next show how the identification of the band structure of a free Hamiltonian defined on ladder allows us to visualize the simple nature of U_σ in the non-interacting model. Defining new Fermi operators $\tilde{c}_{\mu,j,\sigma} = T_{\mu,\alpha} c_{\alpha,j,\sigma}$, $\mu = 1, \dots, \lambda$, with an appropriate $SO(\lambda)$ matrix T , a decoupling of the chains in the free kinetic Hamiltonian can be achieved. Fourier transforming on each channel $c_{\mu,k,\sigma} = \frac{1}{\sqrt{L}} \sum_j e^{ikj} c_{\mu,j,\sigma}$, we get:

$$\begin{aligned} H_t &= -t \sum_{\mu=1}^{\lambda} \sum_{j,\sigma} c_{\mu,j+1,\sigma}^\dagger c_{\mu,j,\sigma} + H.c. - t' \sum_{\mu=1}^{\lambda} \sum_{j,\sigma} \Lambda_\mu n_{\mu,j,\sigma} \\ &= \sum_{\mu=1}^{\lambda} \sum_{k,\sigma} (-2t \cos(k) - t' \Lambda_\mu) n_{\mu,k,\sigma} \end{aligned} \quad (4)$$

where $\Lambda_\mu = 2 \cos(\pi \frac{\mu}{\lambda+1})$. The channels μ can be classified by their parity under the reflection R about the center leg (about the center axis), in the case of odd (even) λ : channels with μ even are odd under R and those with μ odd are even. Furthermore, comparing $\epsilon_\mu(k) = -2t \cos(k) - t' \Lambda_\mu$ with the dispersion relation for the 2D model, we can associate with the band μ the transverse momentum $k_{\perp,\mu} = \pi \frac{\mu}{\lambda+1}$, and form a 2D momentum $\mathbf{k} = (k, k_{\perp,\mu})$. We can now consider twist operators for each channel, $U_{\mu,\sigma} = \exp(2\pi i \sum_j \frac{j}{L} n_{\mu,j,\sigma})$, with $E_{\mu,1} = \langle \phi_0 | U_{\mu,\sigma}^{-1} H_t U_{\mu,\sigma} - H_t | \phi_0 \rangle = O(\frac{1}{L})$, and $TU_{\mu,\sigma}T^{-1} = U_{\mu,\sigma} e^{-2\pi i \nu_{\mu,\sigma}}$, where $\nu_{\mu,\sigma} = \frac{1}{L} \sum_j n_{\mu,j,\sigma}$. Thus, if $\nu_{\mu,\sigma}$ is not an integer, i.e. the band μ is partially filled, a gapless excited state with momentum $2\pi\nu_{\mu,\sigma}$ exists. The action of $U_{\mu,\sigma}$ in momentum space is:

$$U_{\mu,\sigma}^{-1} c_{\mu',k,\sigma'}^\dagger U_{\mu,\sigma} = \begin{cases} c_{\mu',k+\frac{2\pi}{L},\sigma'}^\dagger & \text{if } \sigma = \sigma' \text{ and } \mu = \mu' \\ c_{\mu',k,\sigma'}^\dagger & \text{otherwise} \end{cases}$$

Acting on the non-interacting ground state, $U_{\mu,\sigma}$ creates a new eigenstate of the non-interacting system with the net effect of moving one electron with spin σ and transverse momentum $k_{\perp,\mu}$ from the left to the right Fermi point in the channel μ . Because $\sum_\mu n_{\mu,j,\sigma} = \sum_\alpha n_{\alpha,j,\sigma}$, we have $U_\sigma = \prod_\mu U_{\mu,\sigma}$, and under the action of U_σ , all the fermions with spin σ are shifted one unit to the right. Thus, the excitation created by U_σ in the non-interacting system corresponds to a rigid translation of the Fermi surface, moving an electron from one Fermi

point to the other in each channel, and has a total momentum $\sum_\mu 2k_{F,\mu}$. Notice that, in the limit of a large number of legs, the volume enclosed by the Fermi surface can be expressed as $V_{FS} = \frac{\pi}{\lambda+1} \sum_\mu 2k_{F,\mu}$.

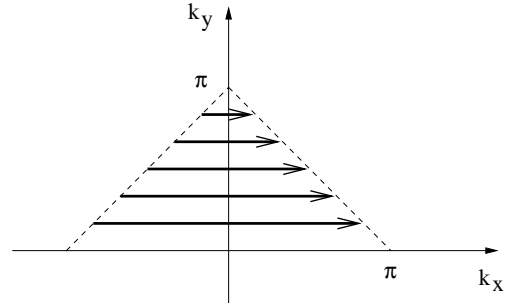


FIG. 2. Global excitation created by U_σ in the free system. The single excitations at the different transverse momenta are shown. The dashed lines join the Fermi points of the channels in a system at half-filling.

As seen before, the global excitation created by U_σ has a vanishing energy in the thermodynamic limit even for the interacting system, but this is in general not the case for those created by a single $U_{\mu,\sigma}$. Hence, in the individual channels, the Fermi wave vectors are not conserved under the interaction, and Fermi points must not necessarily exist. However, the conserved quantity in the interacting system is the sum of the Fermi wave vectors of all the channels, $\sum_\mu 2k_{F,\mu}$, in the sense that the excitation created by U_σ remains gapless. This can be regarded as a generalized Luttinger's Theorem for ladder systems.

To illustrate this point, we consider a 3-chain t-J ladder. The non-interacting band structure has the form of 2 even-parity bands (bonding and anti-bonding) and an odd-parity channel (non-bonding), whose Fermi momenta at half-filling are $\frac{3}{4}\pi$, $\frac{\pi}{4}$, and $\frac{\pi}{2}$ respectively.

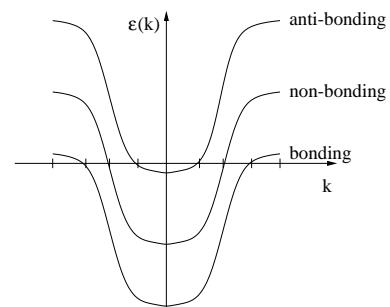


FIG. 3. Free bands for a 3-leg ladder.

Numerical calculations based on exact diagonalization of small clusters [14] show that the ground state for the undoped 3-leg t-J ladder and the ground state for the same system with one hole have opposite parities. A parity -1 is therefore associated with a single hole, and the interpretation, obtained from the investigation of two and more holes, is that below a critical hole density δ_c ,

the doped holes enter the odd parity channel and form a Luttinger Liquid, while the two even parity channels combine to form an insulating spin liquid. Hence, for $\delta = 1 - \nu < \delta_c$, one electron per rung is assigned to each of the two gapped insulating even-parity channels, $\nu_b + \nu_a = 2$, resulting in $2k_F = 2\pi$ for the spin liquid, whereas the remaining $1 - 3\delta$ electrons per rung are assigned to the odd-parity channel, $\nu_{nb} = 1 - 3\delta$, with $2k_F = \pi(1 - 3\delta)$ for the Luttinger liquid. Gapless excitations at $\pi(1 - 3\delta)$ are consistent with our results: $\pi(1 - 3\delta) \equiv \pi(1 - 3\delta) + 2\pi = 3\pi(1 - \delta) = 3\pi\nu$. Thus, the combination with the two even-parity channels in a gapped spin liquid leads to the generalized Luttinger Theorem for a 3-leg ladder: $\sum_{\mu} 2k_{F,\mu} = 3\pi\nu$.

We have seen that the band structure of a non-interacting fermionic ladder system results in momentum states aligned at different transverse momenta parallel to the k_x -axis. We can show that an equivalent construction is possible in other directions in momentum space, starting from ladder structures in other directions than that of the lattice axis. In this case, the generalized LSM Theorem can be applied to show the existence of a low-energy excitation on a λ -leg ladder at momentum $2\pi\lambda\nu_{\sigma}$ in each direction of periodicity of the 2D cubic lattice, if $\lambda\nu_{\sigma}$ is not an integer. Let us consider a ladder G with λ legs defined by a vector $\mathbf{T} = (p, q)$, $p, q \in \mathbf{N}$ coprimes, which joins 2 lattice points. Let \mathbf{n} be a unit vector in the direction of the ladder, and Λ be the quantization length along the ladder direction.

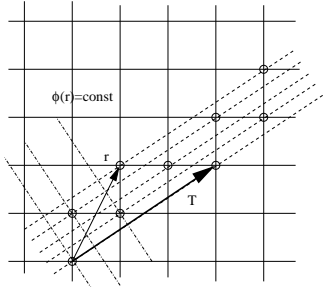


FIG. 4. A $\lambda=5$ -leg ladder in direction \mathbf{T} . The ladder points are defined by the intersection of the diagonal lines with the lattice axis and are denoted by small circles.

A generalized twist operator is $U_{\sigma} = e^{i \sum_{\mathbf{r} \in G} \varphi(\mathbf{r}) n_{\mathbf{r}, \sigma}}$, where $\varphi(\mathbf{r}) = \frac{2\pi}{\Lambda} \mathbf{r} \cdot \mathbf{n}$. By means of an inversion center of G , a parity operator is defined, and an excitation energy of $O(\frac{\lambda}{L})$ for the twisted state can be shown as before.

What can be said in this context about the whole 2D plane? To get insight into the full 2D problem letting the number of the legs going to infinity, we consider $H = -t \sum_{\mathbf{r}, \tau, \sigma} c_{\mathbf{r}+\tau, \sigma}^{\dagger} c_{\mathbf{r}, \sigma} + H_{int}$ where $\mathbf{r} = (j, \alpha)$, $j = 1, \dots, L$, $\alpha = 1, \dots, \lambda$, and the sum over τ extends over next-neighbors: $\tau = \pm \mathbf{e}_x, \pm \mathbf{e}_y$. Periodic boundary conditions are now used for both directions: $c_{\mathbf{r}+\Lambda \mathbf{e}_x + \lambda \mathbf{e}_y, \sigma} = c_{\mathbf{r}, \sigma}$. Obviously, we have again $E_1 = \langle U_{\sigma}^{-1} H U_{\sigma} - H \rangle = O(\frac{\lambda}{L})$.

We see that the thermodynamic limit $\lambda L \rightarrow \infty$ of E_1 does not exist. For example, letting $\lambda, L \rightarrow \infty$ with $\frac{\lambda}{L} \rightarrow 1$, we have $E_1 \sim \text{const}$. This is in agreement with the action of U_{σ} on the ground state for the non-interacting model. In fact, Fourier transforming in a 2D momentum space $c_{\mathbf{k}, \sigma} = \frac{1}{\sqrt{\lambda L}} \sum_{\mathbf{r}} e^{i\mathbf{k} \cdot \mathbf{r}} c_{\mathbf{r}, \sigma}$, we have $U_{\sigma}^{-1} c_{\mathbf{k}, \sigma}^{\dagger} U_{\sigma} = c_{\mathbf{k}+\mathbf{q}, \sigma}^{\dagger}$, with $\mathbf{q} = \frac{2\pi}{L} \mathbf{e}_x$. Hence, the whole Fermi surface is shifted, and even if each single excitation of an electron at a given transverse momentum costs a vanishing energy, the number of such excitations becomes large, as λ increases. We conclude that the LSM Theorem in this form does not allow to show the existence of a low-energy excitation in the whole 2D plane.

In summary, on a ladder with an arbitrary fixed number of legs λ , a low-lying excitation exists at momenta $2\pi\lambda\nu_{\sigma}$ (in each direction of periodicity of the 2D square lattice), if $\lambda\nu_{\sigma}$ is not an integer. The existence of these gapless excitations can be regarded as a statement of the conservation under interaction of the sum of the Fermi wave vectors of the individual channels, corresponding to a generalized Luttinger Theorem for ladder systems.

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