Empirical Asset Pricing

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Liquidity and Asset Pricing

Lecture Outline

1. Liquidity and asset prices
2. Empirical evidence
3. Liquidity risk in other asset classes

Relevant readings:

- Amihud, Yakov, Haim Mendelson, and Lasse H. Pedersen, 2005, Liquidity and asset prices, Foundations and Trends in Finance 1(4), 269–364.
- Pastor and Stambaugh (2003, JPE), Acharya and Pedersen (2005, JFE), Sadka (2006, JFE)
- Franzoni, F., Eric Nowak, and Ludovic Phalippou, 2011, Private equity performance and liquidity risk, SFI working paper

1. Liquidity and asset prices

Asset pricing with frictions

- In frictionless markets, securities with the same cash flow profiles need to have the same price (no arbitrage)
- For no arbitrage to apply, you need to be able to trade at no cost
- If there are trading frictions, it is not necessarily the case that securities with the same cash flows have the same price
- Example: on-the-run vs. off-the-run bonds. Off-the-run bonds trade at a discount
- This means that, with trading frictions, there does not exist a single stochastic discount factor m that prices all securities

The basic model

- Based on Amihud and Mendelson (1986)
- Assume exogenous transaction costs: C^i (cost of selling the asset)
  - We do not specify why transaction costs emerge
  - This topic is studied in microstructure courses
- Assume exogenous trading horizon
  - Individuals live for two periods and need to liquidate security in second period (overlapping generations)
- **Risk neutral individuals**
- Risk free security is perfectly liquid: $R^f = 1 + r^f$
- The (ex-dividend) price at which one buys is: P_t^i

- The proceeds from holding the stock until t+1 are: $d^i + P^i_{t+1} - C^i$
- So, C^i is a bid-ask spread
- We are looking for a stationary equilibrium: $P^i_t = P^i_{t+1} = P^i$
- Because of risk neutrality, the equilibrium condition is

$$P^i = \frac{d^i + P^i - C^i}{R^f} \tag{1}$$

- Solving equation (1), the stationary price is

$$P^i = \frac{d^i - C^i}{r^f} \tag{2}$$

- The price equals the present value of the infinite dividend stream minus the the present value of the infinite stream of transaction costs

- And the *gross expected return* (i.e. before transaction costs) is equal to the the dividend yield because prices are constant

$$E\left(r^i\right) = \frac{d^i}{P^i}$$

- Using equation (2), we obtain an expression for the dividend yield

$$\frac{d^i}{P^i} = r^f + \frac{C^i}{P^i}$$

- Then, the gross expected return becomes

$$E\left(r^i\right) = r^f + \frac{C^i}{P^i}$$

- The expected return is the risk free rate (from risk neutrality) plus an adjustment for the relative transaction cost
- With risk averse investors, there will be a risk premium too
- Prediction of the model: the higher the transaction costs the higher the expected return
- This is not a remuneration for risk, it is a compensation for the *level of illiquidity of the asset*

Sources of illiquidity (sources of C^i)

1. Exogenous trading costs
   - Just think of broker/exchange commissions. Remuneration for setting up a trading system
2. Inventory risk for the market maker
   - Intuition: market maker intermediates between sellers and buyers. Needs to carry inventory. He bears a risk that fundamentals change in the meantime. Bid-ask spread compensates market-maker for inventory risk (Grossman and Miller, 1988)
3. Private information (Adverse Selection component of trading costs)
   - Intuition: if there is a probability of trading against an informed trader, the market maker will require a compensation in the form of bid-ask spread (e.g. Glosten and Milgrom, 1985)
4. Search costs
   - Especially in OTC markets (bonds) investors approach dealers sequentially (search friction). If investors do not have many alternatives to trade, dealers exploit their monopolistic power and set a larger bid-ask spread (Duffie, Garleanu, and Pedersen, 2005)

Time-varying transaction costs: liquidity risk

- This model is based on Acharya and Pedersen (2005)
- Let us assume that transaction costs vary over time

$$C_t = \bar{C} + \rho^C \left(C_{t-1} - \bar{C}\right) + \eta_t$$

the autoregressive process implies some persistence in illiquidity

- Now, bring in risk averse individuals that live for two periods (overlapping generations)
- We are interested in how the security expected return before transaction costs (gross return)

$$E\left(r_t^i\right) = E\left(\frac{d_t^i + P_t^i}{P_{t-1}^i}\right)$$

depends on its relative illiquidity cost

$$c_t^i = \frac{C_t^i}{P_{t-1}^i}$$

the market gross return r_t^M and the market relative illiquidity c_t^M (which is the weighted average of the individual stocks' c_t^i)

- Under standard assumptions CAPM holds for the net returns
  - Intuition: you can think of a frictionless economy with dividends that are $D_t^i - C_t^i$. For this economy, under these assumptions, CAPM holds for net expected returns

$$E_t\left(r_{t+1}^i - c_{t+1}^i\right) = r^f +$$
$$+\lambda_t \frac{cov_t\left(r_{t+1}^i - c_{t+1}^i, r_{t+1}^M - c_{t+1}^M\right)}{var_t\left(r_{t+1}^M - c_{t+1}^M\right)}$$

    where $\lambda_t = E_t\left(r_{t+1}^M - c_{t+1}^M\right) - r^f$

- So, you get a four beta representation for gross expected returns

$$E_t\left(r_{t+1}^i\right) = r^f + E_t\left(c_{t+1}^i\right) + \lambda_t\left(\beta_t^1 + \beta_t^2 - \beta_t^3 - \beta_t^4\right)$$

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$$\beta_t^1 = \frac{cov_t\left(r_{t+1}^i, r_{t+1}^M\right)}{var_t\left(r_{t+1}^M - c_{t+1}^M\right)}$$

is the standard CAPM effect. Then you have three betas that are due to *liquidity risk*

- $$\beta_t^2 = \frac{cov_t\left(c_{t+1}^i, c_{t+1}^M\right)}{var_t\left(r_{t+1}^M - c_{t+1}^M\right)}$$

  – Is positive for most securities

  – Expected returns depend positively on this beta because investors want compensations for assets that become illiquid when the market gets illiquid

- $$\beta_t^3 = \frac{cov_t\left(r_{t+1}^i, c_{t+1}^M\right)}{var_t\left(r_{t+1}^M - c_{t+1}^M\right)}$$

  – Is usually negative. When the market becomes illiquid, that's typically times of low returns

  – Expected returns depend negatively on this beta because investors like securities whose returns increase when the market becomes illiquid

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$$\beta_t^4 = \frac{cov_t\left(c_{t+1}^i, r_{t+1}^M\right)}{var_t\left(r_{t+1}^M - c_{t+1}^M\right)}$$

  - Is usually negative. When the market goes up, that's typically times of high liquidity
  - Expected returns depend negatively on this beta because investors like securities whose transaction costs decrease in down markets

- Further predictions of the model are that:
  - Liquidity predicts future returns (because of persistence of liquidity)

$$\frac{\partial}{\partial C_t^i} E_t\left(r_{t+1}^i - r^f\right) > 0$$

  - Liquidity comoves negatively with contemporaneous returns (as for an expected return shock)

$$cov_{t-1}\left(c_t^i, r_t^i\right) < 0$$

Liquidity risk in the pricing kernel

- Alternatively, one could assume that a liquidity factor appears directly in the pricing kernel (see Bekaert, Harvey, Lundblad 2007)

$$\begin{aligned} m_{t+1} &= -\gamma_w r_{w,t+1} - \gamma_L L_{t+1} \\ \gamma_w &> 0, \quad \gamma_L > 0 \end{aligned}$$

  where $r_{w,t+1}$ is return on the wealth portfolio and L_{t+1} is a factor measuring innovations in aggregate market liquidity

- The authors do not go much into details as to why the liquidity factor appears in the pricing kernel

- They propose two stories:
  - Investors have a preference for liquidity because they are fund managers subject to withdrawals (Vayanos 2004)

- Behavioral stories (Baker and Stein 2004) where high liquidity reflects the large presence of irrational investors in the market and high market valuations (due to short sale constraints). In brief: high $L_{t+1} \rightarrow$ high market valuations $\rightarrow$ Low marginal utility of consumption

- In any case, this pricing kernel generates a multifactor model in which liquidity is one of the factors

2. Empirical Evidence

Measuring liquidity

- Liquidity is a multi-faceted concept: remember the different sources of illiquidity
- Data availability limits researchers' scope:
  - Bid-ask spreads are available mostly for the U.S.
  - Intraday data are available predominantly for the U.S.: Trades and Quotes dataset (TAQ, since 1993)
- Researchers have used different measures of liquidity. Among others (for stocks):

1. The bid-ask spread, typically as a fraction of the price (relative spread)
   - It's a summary measure of transaction costs that captures also asymmetric information and inventory costs
2. Volume (shares or dollars traded over some interval of time) or turnover (volume divided by capitalization)

- Intuition: if a stock is illiquid it is traded less frequently not to incur in higher transaction costs

3. Kyle's (1985) lambda (λ):

   - Kyle's model is based on asymmetric information. Order flow moves prices because uninformed market makers expect to be trading against informed traders

   - Best estimated on intra-day transaction data (TAQ). Regress at the stock level

$$\Delta p_t = \kappa + \lambda q_t + \varepsilon_t$$

   where q_t is signed transaction size (volume). Sign of q_t is positive (buy) if $\Delta p_t > 0$ and sign is negative (sell) if $\Delta p_t < 0$

4. Amihud's (2002) ratio on daily data for day t is

$$ILLIQ_t^i = \frac{|R_t|}{\$Vol_t}.$$

   This quantity captures the return due to a given amount of dollar volume. One can average this variable over a month to get the monthly illiquidity measure

- Intuition: for more illiquid stocks a trade of a given amount has a bigger price impact

5. For international data, Bekaert, Harvey, and Lundblad (2007) use the fraction of days with zero returns within a month
   - In international data you do not have volume
   - Zero return days are typically days of zero volume

- For all measures, liquidity is positively correlated with size and negatively correlated with volatility, which makes identification of a liquidity effect complicated

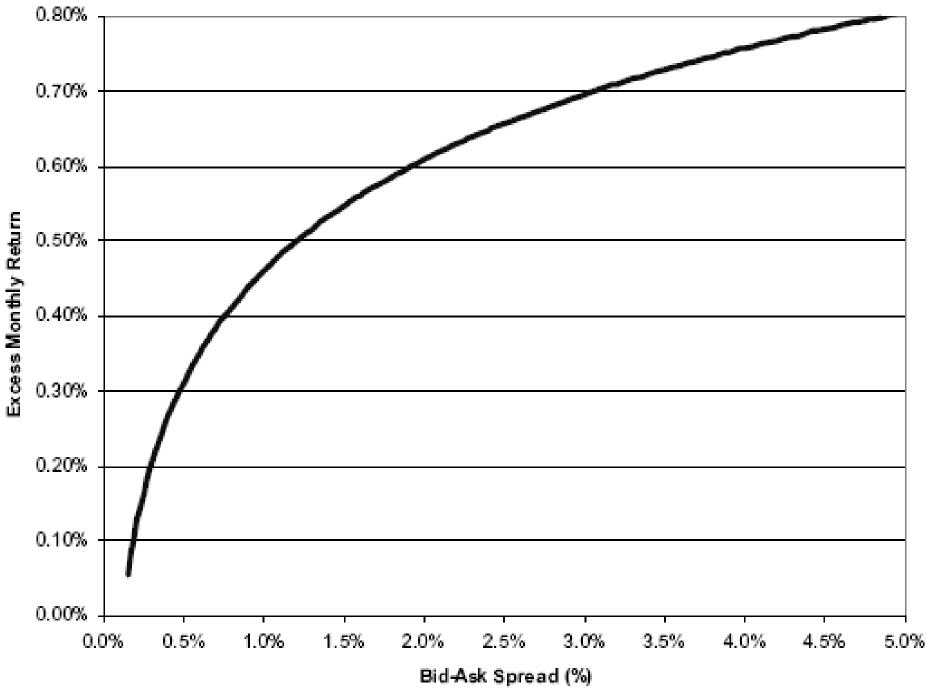
- Also, see Collin-Dufresne and Fos (2015, JF, Do prices reveal the presence of informed trading?)
  - Informed trading is endogenous with respect to market liquidity
  - They show that informed trading (as captured by activist investors' trades reported in Form 13D) is highest when standard measures signal high liquidity (λ, Amihud, etc.)
  - Hence: these measures do not really capture the extent of asymmetric information in the market

Liquidity level effect in the cross-section

- Amihud and Mendelson (1986) use the relative bid-ask spread as a measure of liquidity
- They find that:
  1. Average returns are higher for securities with higher bid-ask spread controlling for beta
  2. The relationship is concave

$$R_j = 0.0065 + 0.0010\beta_j + 0.0021 \ln\left(S_j\right)$$

  where S_j is the relative spread. All coefficients are significant



- Concavity is consistent with **clientele effects** in their model

- That is, more illiquid stocks are held by individuals with longer holding periods. Hence they do not incur in as many trading costs and they require lower compensation for illiquidity

- The effect is confirmed in later studies. But it appears stronger for Nasdaq stocks and in January

Empirical Evidence on Liquidity Risk

Pastor and Stambaugh (JPE 2003)

- In this paper they assume that liquidity is in the pricing kernel and generates a separate factor
- Flavor of ICAPM
- They do not provide a formal model, just an empirical implementation
- Intuition: some institutional investors are subject to solvency/margin constraints and may be forced to liquidate positions when things turn bad
- If liquidation is more likely when aggregate liquidity is low, they will prefer asset whose returns are less sensitive to aggregate liquidity to avoid forced liquidation
- That is, assets whose returns covary positively with aggregate liquidity need to pay a premium
- Example with LTCM in 1998: they invested in long-short positions involving Russian debt. When the Russian crisis hit, their portfolios lost value, they had to liquidate their positions at low prices. LTCM certainly disliked the fact these positions' returns were low when aggregate liquidity was low
- These positions paid out well in the previous three years, suggesting that they were earning a liquidity risk premium

Stock level liquidity

- On daily data (within a month) at the stock level they run

$$r^e_{i,d+1} = \theta_i + \phi_i r_{i,d} + \gamma_i sign\left(r^e_{i,d}\right) v_{i,d} + \varepsilon_{i,d+1}$$

  where $r_{i,d}$ is the daily return, $r^e_{i,d} = r_{i,d} - r_{mkt,d}$, and $v_{i,d}$ is the dollar volume

- Intuition: order flow (signed volume) causes price pressure on day d that is going to partly revert on the next day. The reversion is stronger the more illiquid the stock is. That is, the bigger the liquidity motivated price impact on day d

- The reversion in prices occurs if the price impact is **not due to information**. Therefore, this measure of illiquidity is not related to the information-component of the bid-ask spread

- They expect γ_i to be negative for most stocks

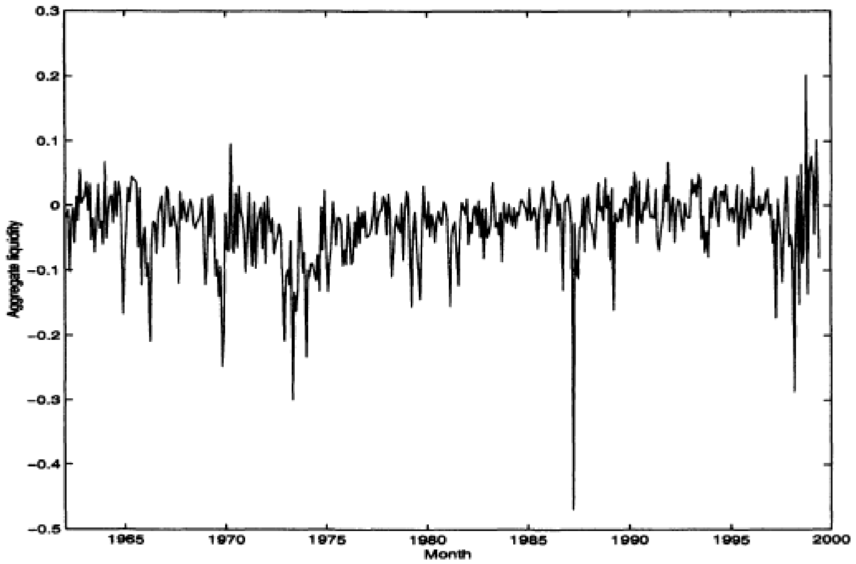
- $\hat{\gamma}_i$ is their measure of stock liquidity

- Evidence of co-movement for $\hat{\gamma}_{it}$ across stocks: systematic variation in liquidity

- For each stock, they get a time series of $\hat{\gamma}_{i,t}$ for each month in which the stock is in the sample
- They compute a market wide measure of illiquidity

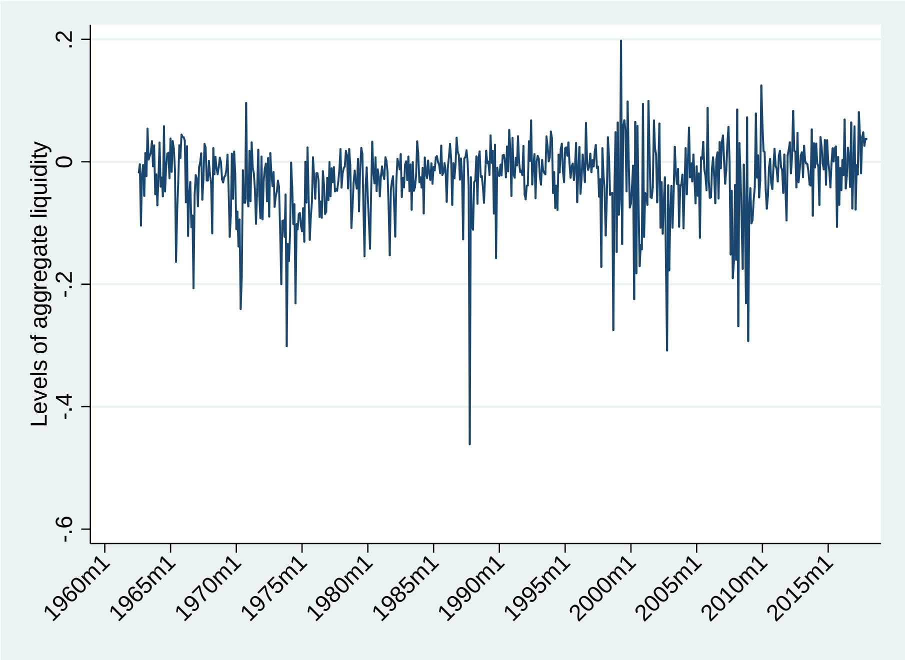
$$\hat{\gamma}_t = \frac{1}{N} \sum_{i=1}^{N} \hat{\gamma}_{i,t}$$

where N is the number of stocks in the sample in month t



- You see that liquidity is low on notable dates. For example: 1987 crash, 1997 Asian crisis, 1998 Russian crisis and LTCM default

- This graph can be updated using the authors' provided series on WRDS. You can then see the drop in liquidity during the 2008 financial crisis



- Finally they compute innovations in $\hat{\gamma}_t$ using an AR(2) model

- The innovation series is the non-traded liquidity factor L_t

Portfolios

- They form 10 portfolios based on betas on the liquidity factor L_t from 4-factor model

$$r_{i,t} = \beta_i^0 + \beta_i^L L_t + \beta_i^M MKT_t + \beta_i^S SMB_t + \beta_i^H HML_t + e_{it}$$

- Properties of portfolios formed on pre-ranking betas:
  - Post-ranking (portfolio) betas align with pre-ranking betas, in the whole sample
  - High liquidity beta stocks are not smaller and less liquid. Different from Acharya and Pedersen (2005)
  - High liquidity beta stocks are momentum winners. Can liquidity explain momentum?

	Decile Portfolio										
	1	2	3	4	5	6	7	8	9	10	10–1
	A. Postranking Liquidity Betas										
Jan. 1966–Dec. 1999	−5.75	−6.54	−4.66	−3.16	.90	−.63	−.86	.68	2.44	2.48	8.23
	(−2.22)	(−2.98)	(−2.59)	(−2.18)	(.69)	(−.54)	(−.68)	(.52)	(1.77)	(1.35)	(2.37)
Jan. 1966–Dec. 1982	−7.28	−8.29	−3.47	−3.15	2.58	−.34	−.47	.73	−2.51	4.19	11.47
	(−1.84)	(−2.54)	(−1.19)	(−1.36)	(1.23)	(−.17)	(−.22)	(.33)	(−1.10)	(1.38)	(2.06)
Jan. 1983–Dec. 1999	−3.00	−4.27	−5.09	−2.36	−1.10	−.84	−1.60	1.94	5.67	.85	3.85
	(−.85)	(−1.37)	(−2.12)	(−1.22)	(−.63)	(−.57)	(−1.06)	(1.22)	(3.23)	(.36)	(.84)
	B. Additional Properties, January 1966–December 1999										
Market cap	2.83	5.90	8.30	7.65	10.67	16.61	15.99	16.02	16.05	14.28	
Liquidity	−.46	−.16	−.10	−.15	−.08	−.07	−.03	−.03	−.04	−.10	
MKT beta	1.24	1.21	1.09	1.05	1.04	1.03	1.00	1.01	.98	.94	−.30
	(37.70)	(44.61)	(48.31)	(56.83)	(62.83)	(68.89)	(62.56)	(60.75)	(55.76)	(40.75)	(−6.85)
SMB beta	.70	.31	.05	.01	−.09	−.12	−.12	−.09	−.12	.05	−.65
	(14.47)	(7.64)	(1.61)	(.26)	(−3.51)	(−5.63)	(−5.04)	(−3.82)	(−4.76)	(1.36)	(−10.14)
HML beta	.07	.19	.23	.20	.11	.14	.08	−.00	−.01	−.34	−.40
	(1.31)	(4.36)	(6.45)	(6.69)	(4.02)	(5.68)	(3.07)	(−.06)	(−.37)	(−9.04)	(−5.74)
MOM beta	−.06	−.10	−.07	−.03	−.03	−.01	.01	−.01	.03	.05	.11
	(−2.43)	(−5.35)	(−4.29)	(−2.19)	(−2.51)	(−.72)	(.53)	(−.72)	(2.72)	(3.02)	(3.41)

Is liquidity risk priced?

- Compute alphas from CAPM and three, or four-factor model (i.e. with momentum) for the ten portfolios

	Decile Portfolio										
	1	2	3	4	5	6	7	8	9	10	10–1
	A. January 1966–December 1999										
CAPM alpha	−5.16	−1.88	−.66	−.07	−1.48	1.48	1.22	1.38	1.68	1.24	6.40
	(−2.57)	(−1.24)	(−.56)	(−.08)	(−1.80)	(1.93)	(1.52)	(1.72)	(1.93)	(1.01)	(2.54)
Fama-French alpha	−6.05	−3.36	−2.15	−1.23	−2.10	.78	.86	1.41	1.90	3.18	9.23
	(−3.77)	(−2.47)	(−1.93)	(−1.37)	(−2.61)	(1.08)	(1.11)	(1.76)	(2.22)	(2.82)	(4.29)
Four-factor alpha	−5.11	−1.66	−1.02	−.76	−1.61	.91	.76	1.55	1.34	2.36	7.48
	(−3.12)	(−1.23)	(−.91)	(−.83)	(−1.96)	(1.22)	(.96)	(1.88)	(1.54)	(2.06)	(3.42)
	B. January 1966–December 1982										
CAPM alpha	−2.26	1.63	.54	.67	−3.09	1.44	.61	1.78	1.43	−.93	1.34
	(−.81)	(.76)	(.31)	(.50)	(−2.69)	(1.29)	(.54)	(1.46)	(1.14)	(−.52)	(.36)
Fama-French alpha	−7.32	−2.22	−1.80	−.75	−3.29	1.03	.20	1.91	2.32	1.18	8.50
	(−3.36)	(−1.23)	(−1.13)	(−.59)	(−2.85)	(.95)	(.17)	(1.56)	(1.86)	(.71)	(2.77)
Four-factor alpha	−6.43	−.25	−.22	−.03	−2.46	1.09	.31	2.89	1.67	−.22	6.21
	(−2.82)	(−.13)	(−.13)	(−.02)	(−2.05)	(.95)	(.25)	(2.28)	(1.28)	(−.13)	(1.95)
	C. January 1983–December 1999										
CAPM alpha	−8.01	−5.33	−1.76	−1.01	.20	1.55	1.74	.70	1.81	3.38	11.39
	(−2.76)	(−2.49)	(−1.08)	(−.77)	(.17)	(1.46)	(1.54)	(.67)	(1.47)	(1.98)	(3.36)
Fama-French alpha	−5.23	−5.08	−2.69	−1.80	−.82	.37	.89	.76	1.25	5.51	10.74
	(−2.23)	(−2.46)	(−1.67)	(−1.41)	(−.72)	(.38)	(.89)	(.72)	(1.05)	(3.51)	(3.53)
Four-factor alpha	−4.43	−3.72	−1.94	−1.52	−.63	.53	.70	.47	.84	5.06	9.49
	(−1.88)	(−1.85)	(−1.21)	(−1.17)	(−.54)	(.54)	(.69)	(.44)	(.70)	(3.20)	(3.12)

- Alphas are negative for low liquidity beta portfolios and positive for high liquidity betas portfolios (annualized)

- The spread portfolio 10-1 earns a significantly positive alpha: 7.5% per year in the 1966-1999 sample

Construct a traded liquidity factor

- Form a long-short position in portfolio 10 minus portfolio 1. These portfolios are either value- or equally-weighted

- You get two traded liquidity factors: LIQ^v, LIQ^e

- You can use the LIQ factors to augment the known factor models and price other assets

- Below, the alphas for the momentum portfolio MOM (long winners and short losers):

	January 1966–December 1999	January 1966–December 1982	January 1983–December 1999
MKT, SMB, HML	16.30	21.65	11.10
	(4.85)	(4.53)	(2.29)
MKT, SMB, HML, LIQ^V	13.89	19.46	8.03
	(4.09)	(4.04)	(1.63)
MKT, SMB, HML, LIQ^E	8.41	16.11	−1.29
	(2.55)	(3.35)	(−.28)

- The liquidity factors substantially reduce the alpha from momentum strategies

- In modern sample with LIQ^e, the alpha from the four-factor model is insignificant

Conclusions

- Aggregate variation in liquidity seems to be a priced risk factor
- The premium for liquidity risk is about 7.5% per year
- The effect of liquidity risk seems to be separate from the liquidity level because high liquidity risk stocks are not necessarily illiquid stocks
- The four-factor model seems to perform well in explaining some anomalies (momentum)

Acharya and Pedersen (JFE 2005)

- We discussed the theoretical model above
- The goal of the paper is to estimate the premiums due to liquidity risk and to the liquidity level
- They can perform this task because they have a fully specified economic model that allows them to disentangle the source of the premiums in returns
- Remember they have four betas plus a premium due to the liquidity level

$$E_t\left(r^i_{t+1}\right) = r^f + E_t\left(c^i_{t+1}\right) + \lambda_t\left(\beta^1_t + \beta^2_t - \beta^3_t - \beta^4_t\right)$$

- $\beta^3 = \frac{cov\left(r^i_{t+1}, c^M_{t+1}\right)}{var\left(r^M_{t+1} - c^M_{t+1}\right)}$ is similar to the liquidity beta of Pastor and Stambaugh (2003), with the opposite sign because Pastor and Stambaugh use a Liqudity factor, while Acharya and Pedersen use an **Illiquidity factor**

Estimates of c_t^i

- They use the Amihud's ratio $ILLIQ_t^i$ to estimate illiquidity at the monthly frequency at the stock level

- This measure is expressed 'percent per dollar'. It is non stationary

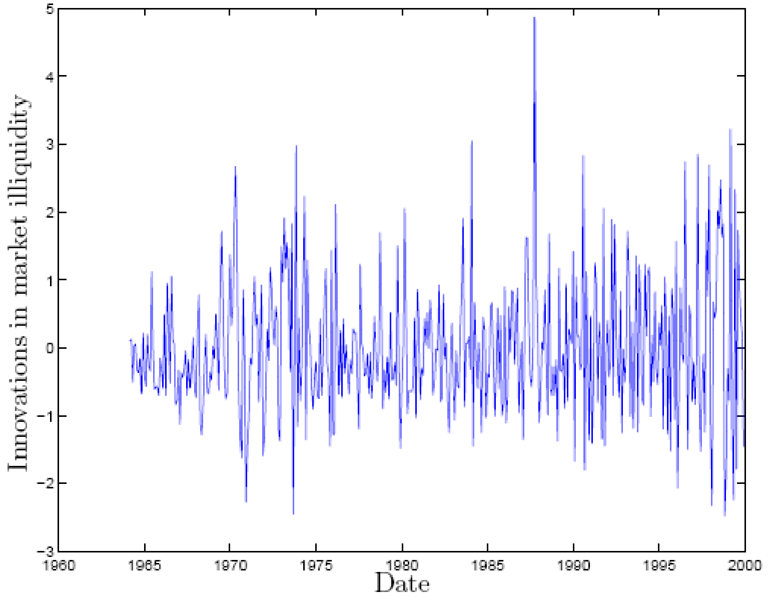
- They normalize it by multiplying by ratio of market cap at $t-1$ to mkt cap in July 1962

- It is a noisy measure of illiquidity. So, they form portfolios and compute the (value- and equally-weighted) averages of c_t^i at the portfolio level

- Liquidity is persistent. Only innovations (news) drive returns. So, they take innovations using an AR(2) model

Estimates of c_t^M

- c_t^M is the average of c_t^i for all stocks
- An AR(2) model is also computed for the market to get innovations in illiquidity



- The spikes correspond to notable liquidity events. For example: 10/1987 (stock market crash), 8/1990 (Iraqi invasion of Kuwait), 4,12/1997 (Asian crisis), 6-10/1998 (Russian default and LTCM crisis)

- Correlation with Pastor and Stambaugh innovations in liquidity is -0.33

The portfolios

- 25 portfolios by prior year illiquidity: c_t^i
- 25 portfolios by volatility of prior year c_t^i : $\sigma\left(c_t^i\right)$

For robustness they also look at:

- 25 size portfolios
- 25 BM and size portfolios

Betas for illiquidity portfolios

	β^{1p} $(\cdot 100)$	β^{2p} $(\cdot 100)$	β^{3p} $(\cdot 100)$	β^{4p} $(\cdot 100)$	$E(c^p)$ (%)	$\sigma(\Delta c^p)$ (%)	$E(r^{e,p})$ (%)	$\sigma(r^p)$ (%)	trn (%)	size (bl$)	BM
1	55.10	0.00	−0.80	−0.00	0.25	0.00	0.48	1.43	3.25	12.50	0.53
	(14.54)	(0.08)	(−5.90)	(−0.10)							
3	67.70	0.00	−1.05	−0.03	0.26	0.00	0.39	1.64	4.19	2.26	0.72
	(16.32)	(0.58)	(−7.14)	(−0.62)							
5	74.67	0.00	−1.24	−0.07	0.27	0.01	0.60	1.74	4.17	1.20	0.71
	(20.44)	(1.27)	(−7.43)	(−1.36)							
7	76.25	0.00	−1.27	−0.10	0.29	0.01	0.57	1.83	4.14	0.74	0.73
	(20.63)	(2.18)	(−7.49)	(−2.03)							
9	81.93	0.01	−1.37	−0.18	0.32	0.02	0.71	1.86	3.82	0.48	0.73
	(33.25)	(3.79)	(−8.00)	(−3.74)							
11	84.59	0.01	−1.41	−0.33	0.36	0.04	0.73	1.94	3.87	0.33	0.76
	(34.21)	(5.07)	(−7.94)	(−5.85)							
13	85.29	0.01	−1.47	−0.40	0.43	0.05	0.77	1.99	3.47	0.24	0.77
	(34.15)	(6.84)	(−8.01)	(−7.46)							
15	88.99	0.02	−1.61	−0.70	0.53	0.08	0.85	2.04	3.20	0.17	0.83
	(42.88)	(6.87)	(−8.35)	(−8.45)							
17	87.89	0.04	−1.59	−0.98	0.71	0.13	0.80	2.11	2.96	0.13	0.88
	(27.54)	(8.16)	(−8.18)	(−9.30)							
19	87.50	0.05	−1.58	−1.53	1.01	0.21	0.83	2.13	2.68	0.09	0.92
	(40.74)	(7.63)	(−8.75)	(−8.77)							
21	92.73	0.09	−1.69	−2.10	1.61	0.34	1.13	2.28	2.97	0.06	0.99
	(37.85)	(7.33)	(−8.34)	(−6.11)							
23	94.76	0.19	−1.71	−3.35	3.02	0.62	1.12	2.57	2.75	0.04	1.09
	(39.71)	(6.85)	(−8.68)	(−5.91)							
25	84.54	0.42	−1.69	−4.52	8.83	1.46	1.10	2.87	2.60	0.02	1.15
	(20.86)	(6.40)	(−8.23)	(−3.35)							

- High correlation among the three liquidity betas, and between liquidity betas and liquidity level across portfolios

- High correlation among liquidity betas is a problem for separate identification of the different effects

- Their liquidity measure c^p correlates with $\sigma\left(c^p\right)$, average returns, turnover, volatility, size, BM

- In general, multicollinearity is a problem in empirical tests

- But we do observe that more illiquid portfolios have higher average returns

Asset pricing tests

- Let us focus on illiquidity and $\sigma(c)$ portfolios. That's where you are more likely to detect an effect of liquidity level and risk

- They first take the predictions of the theory seriously and test the liquidity adjusted CAPM (unconditional version)

$$E\left(r_t^p - r^f\right) = \alpha + \kappa E\left(c_t^p\right) + \lambda \beta^{net,\, p}$$

  where

$$\beta^{net,\, p} = \beta^{1p} + \beta^{2p} - \beta^{3p} - \beta^{4p}$$

  that is, the four betas are constrained to have the same premium

- Main prredictions: 1. $\alpha = 0$ and 2. λ is risk premium on the market

- Also, κ is a coefficient that adjusts for the monthly period in the estimation and typical holding period of an investor

- You can calibrate κ using the average turnover in the sample, 0.034, which corresponds to a holding period of 29 months

Panel A: illiquidity portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.556	0.034					1.512	0.732
	(−1.450)	(—)					(2.806)	(0.732)
2	−0.512	0.042					1.449	0.825
	(−1.482)	(2.210)					(2.532)	(0.809)
3	−0.788		1.891					0.653
	(−1.910)		(3.198)					(0.638)
4	−0.333	0.034	−3.181				4.334	0.843
	(−0.913)	(—)	(−0.998)				(1.102)	(0.836)
5	0.005	−0.032	−13.223				13.767	0.878
	(0.013)	(−0.806)	(−1.969)				(2.080)	(0.861)
6	−0.160		−8.322				9.164	0.870
	(−0.447)		(−2.681)				(3.016)	(0.858)
7	−0.089	0.034	0.992	−153.369	7.112	−17.583		0.881
	(−0.219)	(—)	(0.743)	(−1.287)	(0.402)	(−1.753)		(0.865)
8	−0.089	0.033	0.992	−151.152	7.087	−17.542		0.881
	(−0.157)	(0.166)	(0.468)	(−0.280)	(0.086)	(−1.130)		(0.850)

- Results for illiquidity portfolios:

– Row 1: $\hat{\lambda}$ is positive and significant at 1.5% monthly (high), $\hat{\alpha}$ is not statistically significant (OK), R^2 is high

– Row 2: κ is a free parameter. Not much changes

– In row 3: CAPM has positive λ, but risk premium is too high

– In row 4: CAPM vs Liquidity adjusted CAPM, β^{net} is not significant: not ok!

– But it is significant when looking at $\sigma(c)$ portfolios (see below) or when letting κ be a free parameter (row 5)

– The negative coefficient on β^1 in row 4 is due to the fact that β^1 is also in β^{net} and that the risk premium on the liquidity factors is higher than the risk premium on the market

$$\begin{aligned} E(r_t^p - r_t^f) &= -0.333 + 0.034E(c_t^p) - 3.181\beta^{1p} + 4.334\beta^{net,p} \qquad (27) \\ &= -0.333 + 0.034E(c_t^p) + 1.153\beta^{1p} + 4.334\left(\beta^{2p} - \beta^{3p} - \beta^{4p}\right) \end{aligned}$$

- – In rows 7 and 8, no restriction is imposed on the premia on the different betas. Multicollinearity prevents precise estimation. But fit is good ($R^2 = 88\%$)

Panel B: σ(illiquidity) portfolios

	constant	$E(c^p)$	β^{1p}	β^{2p}	β^{3p}	β^{4p}	$\beta^{net,p}$	R^2
1	−0.528	0.035					1.471	0.865
	(−1.419)	(—)					(2.817)	(0.865)
2	−0.363	0.062					1.243	0.886
	(−1.070)	(2.433)					(2.240)	(0.875)
3	−0.827		1.923					0.726
	(−2.027)		(3.322)					(0.714)
4	−0.014	0.035	−7.113				7.772	0.917
	(−0.037)	(—)	(−1.939)				(2.615)	(0.914)
5	0.094	0.007	−11.013				11.467	0.924
	(0.235)	(0.158)	(−2.080)				(2.480)	(0.914)
6	0.119		−11.914				12.320	0.924
	(0.305)		(−2.413)				(2.608)	(0.917)
7	0.464	0.035	−1.105	−83.690	−74.538	−14.560		0.940
	(0.913)	(—)	(−0.728)	(−0.663)	(−1.175)	(−1.662)		(0.931)
8	0.459	0.148	−1.125	−390.588	−73.552	−21.688		0.942
	(0.565)	(0.140)	(−0.485)	(−0.140)	(−1.943)	(−0.335)		(0.927)

Economic significance of results

- Use the model with β^{net}, that is $\hat{\lambda} = 1.512$ and calibrated $\kappa = 0.034$
- Compare spreads in premia for extreme illiquidity portfolio (port. 25 - port. 1)

$$
\begin{aligned}
\lambda\left(\beta^{2,p_{25}} - \beta^{2,p_1}\right) \cdot 12 &= 0.08\% \\
-\lambda\left(\beta^{3,p_{25}} - \beta^{3,p_1}\right) \cdot 12 &= 0.16\% \\
-\lambda\left(\beta^{4,p_{25}} - \beta^{4,p_1}\right) \cdot 12 &= 0.82\%
\end{aligned}
$$

- Total liquidity risk spread is 1.1%
- Total spread in liquidity level $E\left(c^p\right)$ is 3.5%
- So, total spread due to liquidity (level and risk) is 4.6% per year
- Most important source of liquidity risk is β^4: covariance of stock level transaction costs with market return

- This dimension of liquidity risk is not explored in other studies
- The premium is smaller than what found by Pastor and Stambaugh (2003) who find 7.5% per year
- Possible reasons:
  - P&S sort on liquidity betas and not on liquidity levels
  - P&S they use a different measure of liquidity
  - P&S do not control for $E\left(c^{p}\right)$ in the regressions
  - A&P restrict the λ to be the same for all betas. P&S do not impose restrictions

Conclusions

- Liquidity premium is not huge
- Nice structural model brought to the data
- Problem of multicollinearity between liquidity level and liquidity risk
- Liquidity risk does not solve the value premium (result from BM portfolios)
- Flights to quality or Flights to liquidity: the most illiquid stocks are also those with higher liquidity betas. They suffer most in bad times

Sadka (JFE 2006)

- Momentum has escaped rational explanations so far
- Similarly, the post-earnings-announcement drift (PEAD) is unexplained by known asset pricing models
- Both anomalies involve high turnover to be exploited: they incur high transaction costs
- So, their returns are likely to be sensitive to aggregate variations in liquidity
- Test if liquidity risk explains returns on these strategies

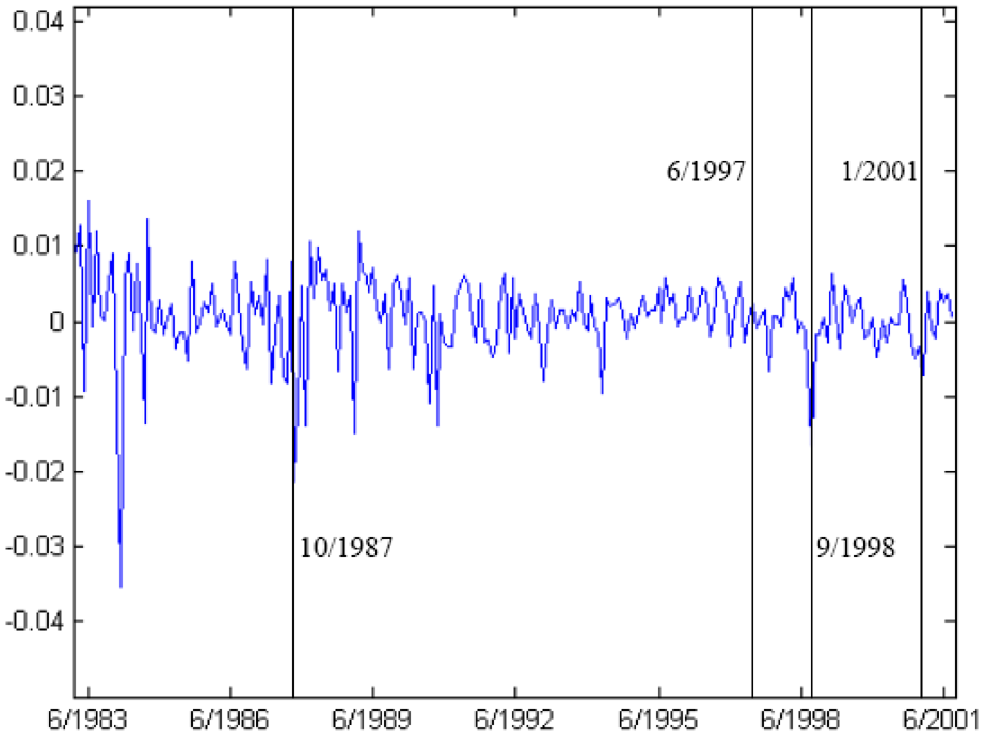
Construction of the liquidity factors

- Estimate the microstructure model of Glosten and Harris (1988) on intra-day transaction data (TAQ)
- Estimate price impact components that are due to:
  - Information asymmetry (permanent impact of order flow on prices)
  - Inventory and order processing (transitory impact of order flow on prices)
- Intuitively:
  - The component due to information has a permanent impact on prices
  - The component due to inventory/order processing has a mean-reverting impact on prices
  - The empirical model breaks down the impact of order flow onto prices into these two components
- Further, for each component, define a:

– Variable cost (linear function of order flow)

– Fixed cost (independent of amount of order flow, it just depends on the sign of the order flow)

- He shows that only innovations in the permanent-variable component represent a priced liquidity factor
- Here it is:



A quick look at the asset pricing tests

- Test assets are: 25 momentum and 25 PEAD portfolios
- Run cross-sectional regressions and see whether the factor has a significant price for risk in the cross-section of portfolio returns

$$E(r_i) = \gamma_0 + \gamma' \beta_i$$

Panel A: MOM Portfolios

	Intercept	MKT	SMB	HML	LIQ ($\bar{\Psi}$)	LIQ (λ)	Adjusted R^2
Premium	1.08	-0.43					0.00
T-Statistic	2.83	-0.76					
Premium	0.62	0.60			-0.32		0.16
T-Statistic	1.14	0.82			-1.66		
Premium	0.62	-0.02				0.47	0.83
T-Statistic	1.42	-0.04				2.21	
Premium	0.54	0.19			-0.03	0.42	0.83
T-Statistic	1.29	0.33			-0.49	2.23	
Premium	1.08	1.29	-1.41	-2.37			0.86
T-Statistic	1.74	1.47	-1.58	-2.38			
Premium	1.09	1.31	-1.36	-2.45	-0.11		0.86
T-Statistic	1.60	1.36	-1.41	-2.14	-1.24		
Premium	0.92	0.87	-0.89	-1.56		0.15	0.87
T-Statistic	1.65	1.15	-1.23	-2.37		0.74	
Premium	0.93	0.88	-0.83	-1.64	-0.07	0.13	0.87
T-Statistic	1.60	1.12	-1.08	-2.44	-1.20	0.66	

Panel B: SUE Portfolios

	Intercept	MKT	SMB	HML	LIQ ($\bar{\Psi}$)	LIQ (λ)	Adjusted R^2
Premium	-2.06	3.27					0.21
T-Statistic	-2.72	3.69					
Premium	-2.55	3.69			0.08		0.20
T-Statistic	-2.86	3.70			1.29		
Premium	-3.18	4.19				1.03	0.60
T-Statistic	-1.96	2.41				2.35	
Premium	-3.10	4.12			0.06	1.04	0.58
T-Statistic	-1.81	2.27			0.55	2.34	
Premium	-1.00	2.94	-2.32	-1.04			0.41
T-Statistic	-1.02	2.96	-2.32	-1.74			
Premium	-1.29	3.20	-2.09	-1.14	0.01		0.39
T-Statistic	-1.26	3.06	-2.14	-1.90	0.09		
Premium	-2.44	3.85	-0.66	-1.35		0.82	0.62
T-Statistic	-1.41	2.33	-0.44	-1.48		2.24	
Premium	-2.37	3.79	-0.69	-1.32	0.02	0.83	0.60
T-Statistic	-1.36	2.27	-0.47	-1.46	0.25	2.24	

- The permanent-variable liquidity factor LIQ^{λ} is priced in most specifications, and also with the three Fama-French factors for SUE portfolios

- To compute the spread in returns due to liquidity risk use

$$\hat{\lambda}_{LIQ}\left(\beta^{25}_{LIQ} - \beta^{1}_{LIQ}\right)$$

- This is 6.5% for momentum and 6.8% for SUE portfolios per year

- These premiums can explain a large fraction of the momentum returns (23.16% per year) and even larger for SUE returns (9.12% per year)

Conclusions

- Time variation in the component of liquidity that is information related explains returns on momentum and SUE portfolios
- This component of liquidity relates to the relative weights of informed traders and noise traders in the market
- Sadka argues that momentum and SUE returns are information related
- Intuition: they depend on reaction to information events (e.g. earnings announcements)
- Your strategy involves trading stocks for which there were recent releases of information and hoping that prices continue to move with the trend (momentum)
- So, you are doing uninformed trading
- If there is large information asymmetry in the market, your strategy is more likely to underperform because you may trade against somebody who knows more than you

- That means that these strategies are exposed to information-related liquidity risk. As such, they need to pay a premium

- Main contribution of the paper: characterize the dimension of liquidity (information asymmetry) that is priced

3. Liquidity risk in other assets

Background

- Several papers find that liquidity risk is priced in other asset classes
- International equity markets: Bekaert, Harvey, and Lundblad (2007)
- Bond markets: Beber, Brandt, and Kavajecz (2008), Chordia, Sarkar, and Subrahmanyam (2005), Li, Wang, Wu, and He (2009) and Acharya, Amihud, and Bharath (2010),
- Credit derivative markets: Longstaff, Mithal, and Neis (2005), Bongaerts, de Jong, and Driessen (2011), and Longstaff, Pan, Pedersen, and Singleton (2007)
- Hedge funds: Sadka (2009)
- Boyson, Stahel, and Stulz (2010) show that there is an increase in correlation across different hedge fund styles at times of low aggregate liquidity
- The recent financial crisis has pointed out the role of deteriorating liquidity in generating correlation across asset classes

Franzoni, Nowak, Phalippou (2012, JF)

- Liquidity risk in private equity (PE) returns
- PE is an alternative asset class that hovers around $3 trillion of assets under management
- Very popular in recent years
- PE investments are part of a diversified portfolio of large institutional investors (pension funds, endowments, etc.)
- However, portfolio diversification can be a illusory if a common liquidity factor moves returns of different asset classes
- Especially in bad times, such as the last financial crisis, when liquidity dries up
- PE is likely to be subject to aggregate liquidity shocks:
  - Large transactions in entire companies. Likely to generate high transaction costs and need for a liquid market

- High leverage that needs to be refinanced. If liquidity in credit markets shuts down, private equity funds are obliged to liquidate their investments

- Question: do PE returns load on liquidity factor?

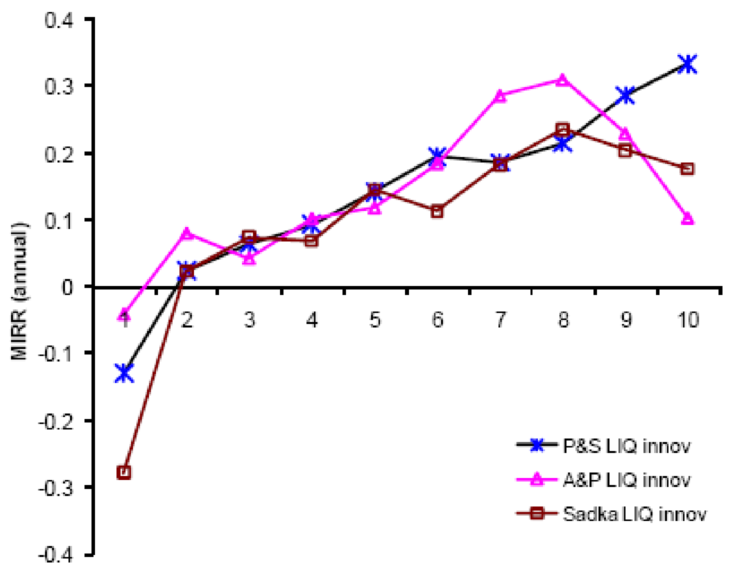
- To what extent a liquidity risk premium can explain the high returns on PE

What is private equity?

- A private equity fund is a pooled investment vehicle that raises capital from large retail investors and especially institutions (pension funds, endowments, insurance companies) with the purpose of investing it over the course of 10 years in a portfolio of companies
- The fund typically purchases distressed companies, brings them private (if they are public), restructures them, and sells them back to the market via IPO or trade sale
- Most common form of investment is leveraged buyout (LBO): high degree of leverage (typically 90% at the start)
- The PE funds are structured as Limited Partnerships. The fund investors are *limited partners*. They do not control the investment process
- The funds are started by PE firms (e.g. KKR, Blackstone), whose representatives retain the role of managing partners in the funds (*general partners* who manage the operations)
- The fund pays dividends to limited partners and management and performance fees (*carried interest*) to general partners (2-20 fee structure)
- There is hurdle rate (often 8%) below which no performance fees are paid

Sentivity to aggregate liquidity

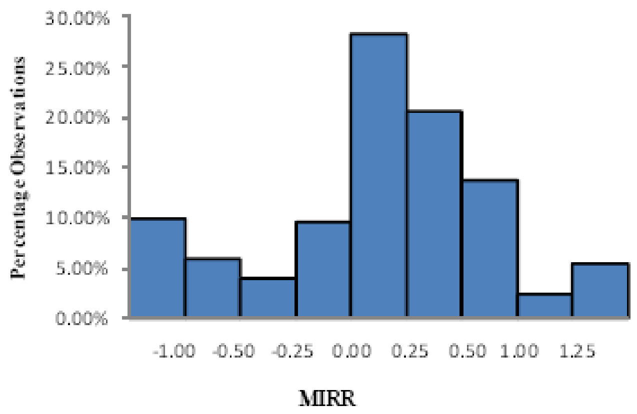
- In our data set, thanks to a private data provider (CEPRES), we have cash flows on each investment in a fund, for many funds
- We compute the investment returns as a modified internal rate of return (MIRR) assuming that intermediate cash flows are reinvested in the S&P500
- We show that the returns on PE are significantly related to aggregate liquidity conditions as measured by different factors (Pastor and Stambaugh, Acharya and Pedersen, Sadka)



- Compute average realization of liquidity factor during the life of investment
- For example: investments held during the decile 1 of liquidity conditions according to P&S have average annual return of -12%. Investments held during decile 10 of P&S have return of 32% annually
- We show that the relation between PE return and liquidity factors is significant controlling for macroeconomic conditions and other known asset pricing factors
- Moreover, the relation between PE returns and the liquidity factor is explained by the tightening of the credit supply (measured by the Federal Reserve Survey of Senior Loan Officers)
- Intuition: supply of credit affects ability to refinance loans and, hence, performance of PE investments
- Also: supply of credit (funding liquidity) is related to market liquidity through the ability of arbitrageurs to participate in the market (see Brunnermeier and Pedersen 2009)

Betas on the factors

- PE returns are highly non-normal because of high-fraction of bankrupt deals (MIRR=-1)



- This issue would make inference unreliable
- Then, we form portfolios based on the month when the investment is started
- We treat the portfolio returns as a time-series of returns on the whole asset class private equity
- We estimate alphas and betas using one regression of this series of (portfolio) returns on the series of average factor realizations over the portfolio life

- Here are the betas for the different factor model specifications

Panel A: Risk Models			
Model:	Market	FF	PS
IML_PS			0.678
			(3.238)
Rm-Rf	1.001	1.526	1.299
	(6.083)	(4.798)	(4.166)
HML		0.596	0.93
		(1.776)	(2.761)
SMB		0.071	0.034
		(0.252)	(0.127)
Constant	0.005	-0.001	-0.002
	(4.784)	(-0.262)	(-0.626)
Sigma	0.046	0.045	0.043
N	103	103	103

- PE loads significantly on the MKT, HML, and P&S traded liquidity factor

Cost of capital and alpha

- Using the average factor realization in the sample, we can compute the component of the cost of capital due to each factor
- Also, we can compute the alpha, that is, the part of the average return that is unexplained by the factors

Panel B: Alpha, Risk Premium, and Cost of Capital			
Model:	Market	FF	PS
Total Risk premium	7.705%	15.006%	17.973%
	(6.129)	(3.926)	(4.801)
Risk premium components:			
$\beta_{liq} \times \mu_{liq}$			3.110%
			(3.315)
$\beta_{mkt} \times \mu_{mkt}$	7.705%	11.753%	10.013%
	(6.129)	(4.874)	(4.262)
$\beta_{hml} \times \mu_{hml}$		3.043%	4.750%
		(1.811)	(2.828)
$\beta_{smb} \times \mu_{smb}$		0.210%	0.101%
		(0.257)	(0.131)
Risk free rate (in sample)	5.816%	5.816%	5.816%
Cost of Capital (in sample)	13.521%	20.822%	23.790%
	(10.755)	(5.447)	(6.355)
Alpha	8.545%	2.291%	0.181%
	(8.679)	(0.760)	(0.059)

- Liquidity risk has premium of about 3%, the MKT pays 10%, and HML about 4.7%

- The total cost of capital of PE is around 23.7% (component due to risk free rate is 5.8% in the sample)

- Important: the alpha of PE goes to zero when the liquidity factor is included!

Conclusions

- First paper to show that PE loads on a liquidity risk factor
- Important in terms of performance of PE in extreme events: the portfolio diversification allowed by private equity vanishes in bad times
- We show that compensation for liquidity risk is about 3%
- Relevant from the point of view of PE investors for the computation of the cost of capital and risk management (i.e. PE loads on extreme liquidity events)
- Finally, compensation for liquidity risk can account for abnormal performance of PE (alpha=0)