Empirical Asset Pricing

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Cross-Sectional Anomalies: The Debate

Lecture Outline

1. Fama and French's results: size and B/M
2. Other anomalies: reversal and momentum
3. Daniel and Titman reply to FF
4. Idiosyncratic volatility is (inversely) priced
5. Betting against beta
6. Investment-based asset pricing

Relevant readings:

- Fama and French, 1992, "The Cross-Section of Expected Stock Returns", Journal of Finance
- Fama and French, 1993, "Common Risk Factors in the Returns on Stocks and Bonds", Journal of Financial Economics

- Cochrane, section 20.2
- Daniel and Titman, 1997, "Evidence on the Characteristics of Cross-Sectional Variation in Stock Returns", Journal of Finance
- Ang, Hodrick, Xing, and Zhang, 2009, "High Idiosyncratic Volatility and Low Returns: International and Further U.S. Evidence", JFE
- Frazzini and Pedersen, 2014, "Betting against Beta", Journal of Financial Economics
- Hou, Xue, Zhang, 2015, "Digesting anomalies: An investment approach", RFS
- Also: take a look at the other papers cited in these class notes

1. Fama and French: Size and B/M

Previous Studies

- A number of studies pointed out deviations from CAPM
- Size effect (Banz, 1981): small stocks, high average returns
- Leverage (Bhandari, 1988): high leverage, high average returns
- Book-to-Market (B/M): B/M positively related to average returns (Stattman, 1980)
- Earnings-to-Price (E/P): higher E/P, higher average returns (Basu, 1983)
- Price-ratio should capture expected returns. Think of Gordon's model:

$$P = \frac{D}{R - G}$$

- Still, if CAPM works, beta should suffice to explain expected returns

Fama and French, 1992, Journal of Finance

- All these price ratios cannot be jointly significant: they capture similar effects
- Goal of the paper: see which ones are really relevant

Results:

1. Beta has no role in explaining cross-section of returns in 1963-1990: clear rejection of CAPM
2. Size, E/P, B/M, Leverage are all individually significant in univariate tests. But in multivariate tests, only Size and especially B/M are significant

Data and Methodology

- All (non-financial) firms in the intersection of CRSP and Compustat: 1963-1990
- Match accounting data of December year $t-1$ to returns from July of year t to June of year $t+1$. Goal: allow some time for information to spread
- Fama and MacBeth regressions: regress monthly returns on betas and accounting variables
- Important Issue: computation of betas
  - Need portfolios to reduce measurement error in betas
  - But accounting variables are well measured at firm level
  - Then: assign to each stock the beta of a portfolio to which it belongs in a given period and run regressions at firm level

- Details:
  - Form 10 size groups by the deciles of the size (market capitalization) distribution using NYSE breakpoints
  - Not enough: there is correlation between size and beta, and you want independent variation by beta for efficiency
  - In each size group, form ten portfolios by pre-ranking betas (5 years of returns before July of year t)
  - You get 100 portfolios. Compute post-ranking betas on whole sample for each portfolio (using Dimson adjustment). Assign to each security the post-ranking beta
- Portfolio grouping could reduce dispersion of betas and power of tests

Regression Results

- The cross-sectional regression they run on each month of data is

$$R_{it} = \lambda\beta_{it} + \gamma z_i + u_{it} \qquad i = 1...N$$

  where β_{it} are estimated in a first-pass as described above, and z_i contains different combinations of the variables: size, B/M, E/P, Asset/M, Asset/B

- Get final estimates as time-series averages of monthly estimates. Use standard error of the mean

- Results in Table III

β	ln(ME)	ln(BE/ME)	ln(A/ME)	ln(A/BE)	E/P Dummy	E(+)/P
0.15 (0.46)						
	−0.15 (−2.58)					
−0.37 (−1.21)	−0.17 (−3.41)					
		0.50 (5.71)				
			0.50 (5.69)	−0.57 (−5.34)		
					0.57 (2.28)	4.72 (4.57)
	−0.11 (−1.99)	0.35 (4.44)				
	−0.11 (−2.06)		0.35 (4.32)	−0.50 (−4.56)		
	−0.16 (−3.06)				0.06 (0.38)	2.99 (3.04)
	−0.13 (−2.47)	0.33 (4.46)			−0.14 (−0.90)	0.87 (1.23)
	−0.13 (−2.47)		0.32 (4.28)	−0.46 (−4.45)	−0.08 (−0.56)	1.15 (1.57)

- Row 1: weak explanatory power of beta (insignificant risk premium)
  - Note: on more recent data, the slope on beta is even negative

- There are size and B/M effects, even controlling for beta (rows 2, 3, and 4)
- B/M prevails on size effect (row 7)
- E/P effect subsumed by B/M
- Leverage: yes, with market leverage; no, with book leverage. The two coefficients are symmetric: it's really a B/M effect

$$c\,\mathsf{log}(A/M) - c\,\mathsf{log}(A/B) = c\,\mathsf{log}(B/M)$$

Major Issue

- Beta is imperfectly measured. So, explanatory power of beta captured by other variables that are correlated with beta (for example: size) and are measured more precisely

- F&F replies:
  - Post-ranking beta is highly correlated with pre-ranking beta
  - No reason for beta insignificant alone (indeed: measurement error would be a reason!)

- True puzzle is more evident today: beta is negatively correlated with B/M. Low beta stocks have high average returns. Opposite of CAPM!

Suggested Interpretations

1. Rational Story:
   - Size and B/M proxy for underlying risks which are associated with the behavior of earnings of small and value firms
   - Chan and Chen (1991, JF) show that small firms have worse performance under several profitability measures. Some of them are 'fallen angels'
   - F&F (1995, JF) show that there are size and value factors in earnings too
   - Argument for 'distress risk factor' (ICAPM story)

2. Behavioral Story:

- Correction of previous over-reaction (mean-reversion)
- Idea: some firms indeed display negative earnings for a number of periods $\longrightarrow$ investors over-react and drive prices down too much because they incorrectly expect continuation of bad performance (B/M $\uparrow$). But then earnings mean-revert and investors are positively surprised: P $\uparrow$ and $R > 0$. Vice versa for growth stocks
- This is the view of Lakonishok, Shleifer and Vishny (LSV, 1994, JF), and La Porta and LSV (1997, JF)

Fama and French, 1993, JFE

- Their previous paper shows that Size and B/M matter in explaining the cross-section of returns
- Risk factors or characteristics?
- If risk factors, need to show covariation of size and B/M portfolios with asset returns
- To provide evidence of comovement of returns, need to use time series approach
- In time series approach, a test of the AP model is a test that the alphas are equal to zero
- Need the factors to be excess returns

Structure of the paper

- The paper aims at empirically defining a unique AP model that explains returns on stocks and bonds
- They use bond market factors (default and term premia) and stock market factors (the market index, HML, and SMB)
- They find that bond factors explain bond returns and stock factors explain stock returns
- The difference in returns between the two markets is captured by the loading on the market factor
- However, what people take away from this paper is the three-factor model for the equity market. That's our focus

The three factors

- One factor is the excess return on the market: $R_{m,t} - R_f$
- Two other factors come from double sorting of stocks along size and B/M dimensions:
  - 2 groups by size: Small, Big (NYSE breakpoints)
  - Independently, 3 groups by B/M: High, Medium, Low
  - Form 6 value-weighted portfolios from intersection
  - Then:

$$
\begin{aligned}
HML &= \frac{1}{2}(HS + HB) - \frac{1}{2}(LS + LB) \\
SMB &= \frac{1}{3}(LS + MS + HS) \\
&\quad -\frac{1}{3}(LB + MB + HB)
\end{aligned}
$$

- HML ideally captures effect B/M controlling for size. SMB captures effect of size controlling for B/M
- The three factors are excess returns. Hence, you can use time-series asset pricing tests

The test assets

- Twenty-five value-weighted portfolios: 5$\times$5 sorting by size and B/M
- Like before, B/M is measured in December of year $t-1$, size is measured in June of year t, and portfolios are formed annually in July of year t
- Sample: 1963-1991
- Summary statistics: average (monthly) returns increase with B/M and decrease with size

$$\begin{aligned} \bar{R}^e_{s=1,bm=5} &= 1.01\% \\ \bar{R}^e_{s=5,bm=1} &= 0.40\% \end{aligned}$$

A tautology?

- Posibble doubt: explain B/M and size sorted portfolios using B/M and size sorted factors. Is it not a tautology?
- No!
  - Think of this example: form portfolios by the stock's first letter (A, B, C, ..., Z)
  - Then, form zero-investment portfolio (excess return) by going long in A-M stocks and short in N-Z stocks
  - Do you expect to find any significant beta on this portfolio?
  - No, because covariation emerges only if there is common factor in returns. Suppose common factor in returns:

$$
\begin{aligned}
R_{it} &= \beta_i f_t + u_{it} \\
Var\left(u_{it}\right) &= \sigma_u^2 \\
Cov\left(u_{it}, f_t\right) &= 0 \\
Cov\left(u_{it}, u_{jt}\right) &= 0
\end{aligned}
$$

Consider a portfolio P based on N securities

$$\begin{aligned} & Cov\left(R_{it}, R_t^P\right) \\ = \; & Cov\left(\beta_i f_t + u_{it}, \frac{1}{N}\sum_{j=1}^{N}\left(\beta_j f_t + u_{jt}\right)\right) \\ = \; & \beta_i \bar{\beta} \sigma_f^2 + \frac{1}{N}\sigma_u^2 \end{aligned}$$

the second term disappears as the number of stocks in the portfolio increases ($N \to \infty$). If there is no common factor ($\sigma_f = 0$), then there is no covariation

Results on covariation

- Look at loadings on factors and R^2 from time-series regressions
- Compare one-factor (CAPM, Table 4) and three-factor model (Table 6)

Table 4

Regressions of excess stock and bond returns (in percent) on the excess stock-market return, $RM-RF$: July 1963 to December 1991, 342 months.[a]

$$R(t) - RF(t) = a + b[RM(t) - RF(t)] + e(t)$$

Dependent variable: Excess returns on 25 stock portfolios formed on size and book-to-market equity

	Book-to-market equity (BE/ME) quintiles									
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	b					$t(b)$				
Small	1.40	1.26	1.14	1.06	1.08	26.33	28.12	27.01	25.03	23.01
2	1.42	1.25	1.12	1.02	1.13	35.76	35.56	33.12	33.14	29.04
3	1.36	1.15	1.04	0.96	1.08	42.98	42.52	37.50	35.81	31.16
4	1.24	1.14	1.03	0.98	1.10	51.67	55.12	46.96	37.00	32.76
Big	1.03	0.99	0.89	0.84	0.89	51.92	61.51	43.03	35.96	27.75
	R^2					$s(e)$				
Small	0.67	0.70	0.68	0.65	0.61	4.46	3.76	3.55	3.56	3.92
2	0.79	0.79	0.76	0.76	0.71	3.34	2.96	2.85	2.59	3.25
3	0.84	0.84	0.80	0.79	0.74	2.65	2.28	2.33	2.26	2.90
4	0.89	0.90	0.87	0.80	0.76	2.01	1.73	1.84	2.21	2.83
Big	0.89	0.92	0.84	0.79	0.69	1.66	1.35	1.73	1.95	2.69

- $R^2 \approx 61\% - 92\%$
- Value stocks have low betas compared to growth stocks, but higher average returns (value premium)

Size quintile	Book-to-market equity Low	2	3	4	High
			b		
Small	1.04	1.02	0.95	0.91	0.96
2	1.11	1.06	1.00	0.97	1.09
3	1.12	1.02	0.98	0.97	1.09
4	1.07	1.08	1.04	1.05	1.18
Big	0.96	1.02	0.98	0.99	1.06
			s		
Small	1.46	1.26	1.19	1.17	1.23
2	1.00	0.98	0.88	0.73	0.89
3	0.76	0.65	0.60	0.48	0.66
4	0.37	0.33	0.29	0.24	0.41
Big	−0.17	−0.12	−0.23	−0.17	−0.05
			h		
Small	−0.29	0.08	0.26	0.40	0.62
2	−0.52	0.01	0.26	0.46	0.70
3	−0.38	−0.00	0.32	0.51	0.68
4	−0.42	0.04	0.30	0.56	0.74
Big	−0.46	0.00	0.21	0.57	0.76

	R^2				
Small	0.94	0.96	0.97	0.97	0.96
2	0.95	0.96	0.95	0.95	0.96
3	0.95	0.94	0.93	0.93	0.93
4	0.94	0.93	0.91	0.89	0.89
Big	0.94	0.92	0.88	0.90	0.83

- R^2 increases substantially with three-factor model: **evidence of covariation of returns with two new factors**

- $R^2 \approx 83\% - 97\%$ almost complete spanning (APT!?)

- Factor loadings:
  - on the market $b \approx 1$ for all assets. It's what you'd expect, because the market factor captures orthogonal variation in returns of size and B/M sorted portfolios after controlling for component due to exposure to long-short (i.e., almost zero-beta) size and B/M factors
  - loading on HML, $h > 0$ for value and $h < 0$ for growth
  - loading on SMB, $s > 0$ for small and $s < 0$ for big

Results on AP performance

- The question is whether the model explains returns entirely ($\alpha_i = 0, \forall i$)
- Compare intercepts from CAPM (Table 9a, Panel (ii)) and the three-factor model (Table 9a, Panel (iv))

	Book-to-market equity (BE/ME) quintiles									
	a					$t(a)$				
Size quintile	Low	2	3	4	High	Low	2	3	4	High
	(ii) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + e(t)$									
Small	−0.22	0.15	0.30	0.42	0.54	−0.90	0.73	1.54	2.19	2.53
2	−0.18	0.17	0.36	0.39	0.53	−1.00	1.05	2.35	2.79	3.01
3	−0.16	0.15	0.23	0.39	0.50	−1.12	1.25	1.82	3.20	3.19
4	−0.05	−0.14	0.12	0.35	0.57	−0.50	−1.50	1.20	2.91	3.71
Big	−0.04	−0.07	−0.07	0.20	0.21	−0.49	−0.95	−0.70	1.89	1.41
	(iv) $R(t) - RF(t) = a + b[RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$									
Small	−0.34	−0.12	−0.05	0.01	0.00	−3.16	−1.47	−0.73	0.22	0.14
2	−0.11	−0.01	0.08	0.03	0.02	−1.24	−0.20	1.04	0.51	0.34
3	−0.11	0.04	−0.04	0.05	0.05	−1.42	0.47	−0.47	0.71	0.56
4	0.09	−0.22	−0.08	0.03	0.13	1.07	−2.65	−0.99	0.33	1.24
Big	0.21	−0.05	−0.13	−0.05	−0.16	3.27	−0.67	−1.46	−0.69	−1.41

- Size and B/M effects in CAPM: significantly positive alphas for small and especially value portfolios. For example, $\alpha_{s=1,bm=5} = 0.54$ (t-stat=2.53)

- Instead, with three-factor model alphas of small and value portfolios are closer to zero
- Still, some problems with small-growth (1,1) portfolio
- According to F&F these are less liquid stocks: no model would work
- Test for joint significance of the alphas (GRS statistic): Table 9c (column (ii) is CAPM, column (iv) is three-factor model)

	Regression (from tables 9a and 9b)				
	(i)	(ii)	(iii)	(iv)	(v)
F-statistic	2.09	1.91	1.78	1.56	1.66
Probability level					
Bootstrap	0.998	0.996	0.985	0.951	0.971
F-distribution	0.999	0.996	0.990	0.961	0.975

- The table shows $1 - Pvalue$. Hence, all the asset pricing models are rejected
- F&F don't make a big deal out of it. They argue that rejection is due to good fit of their model which reduces residual variance and increases power ($\Sigma \downarrow$ and $\alpha'\Sigma^{-1}\alpha \uparrow$: rejection due to variance and not to level of α)
- Remember Cochrane's point: rejection or acceptance of an AP model rarely comes out of purely statistical tests

Interpretation

- F&F conclude that B/M and Size factors represent a priced source of risk
- Arguably, their model is empirically motivated (ad hoc!)
- Future work needs to find sources of economic risk behind these factors
- They propose their model as the new standard in asset pricing

What are size and B/M factors (rational stories)?

- F&F argue there is a ‘distress risk factor’ behind size and B/M
- Indeed, stocks become value (high B/M) after a string of bad news
- Being on the verge of bankruptcy, they are more exposed to shocks
- However, this has to be **aggregate** distress. Idiosyncratic distress can be diversified away
- So, they have to be stocks that are particularly at risk, when the economy is doing poorly
- In Merton’s ICAPM framework, they are poor hedges for variables that affect investors marginal utility: i.e. they underperform when investment opportunities are expected to deteriorate
- Moreover, this riskiness must be partly orthogonal to market risk, else the market factor would be enough

- Consistent with ICAPM: Liew and Vassalou (2000, JFE) show that HML and SMB positive correlate with GDP growth, even controlling for market factor

- Heaton and Lucas (2000, JF) show that the average investor is an owner of a small firm subject to distress risk. Rationally, he should shy away from the risks of small and value firms ('background risk' story)

- Lettau and Ludvigson (2001, JPE) instead argue that the market beta of value stocks increases in bad times (conditional CAPM story for value premium)

- Alternative interpretation is APT:
  - Three-factor model explains almost 100% of return variation
  - By arbitrage arguments, the expected return on these assets must be a combination of return on factors
  - Said differently, given the failure of CAPM to explain average returns, there would be arbitrage opportunities, if the returns on these mispriced stock did not have additional factors in their CAPM residuals (i.e., one could form well-diversified portfolios, get rid of any risk, and enjoy the premia)
  - But in a world where the law-of-one-price holds, arbitrage opportunities are not possible. Hence, there have to be additional factors spanning returns

2. Other anomalies: reversal and momentum

Fama and French, 1996, JF

- F&F show that the three-factor model works with other 'anomalies':

  1. CF/P portfolios
  2. E/P portfolios
  3. Past sales growth portfolios
  4. Long-term past returns (reversal) portfolios

- It does not work, however, with:

  5. Short-term past returns (momentum) portfolios

- It is evident that the tests assets in 1 to 3 are not really different. Notice that high E/P and CF/P stocks, like high B/M firms, have had a string of bad news
- A similar story applies to Reversal portfolios
- In the end, except for momentum, these are not true out-of-sample tests of the model

Reversal

- De Bondt and Thaler (1995, JF) form portfolios on the basis of past 3 to 5 year returns
- They show that long term losers outperform long term winners in the next 3 to 5 years
- A string of bad news gives a low price and a high B/M
- Hence, given the value premium, a stock with low past returns should earn high returns in the future
- In the end, the reversal anomaly is really related to the B/M anomaly

Momentum

- Jegadeesh and Titman (1993, JF) show that trading strategies based on short term losers and winners generate superior returns
- **Cross-sectional** anomaly
- Specifically, each month, buy stocks that are winners (in the past 3 to 12 months, skipping one month) and hold them for the next 3 to 12 months. Symmetrically, sell the short-term losers. On average, the abnormal return is about 1% monthly (1965-1989)
- Jegadeesh and Titman (2001, JF) find persistence of these returns in later sample
- This anomaly is not explained by the F&F three-factor model (F&F, 1996, JF). It eludes risk-based explanations
- Korajczyk and Sadka (2004, JF) show that:
  - The profits survive after accounting for proportional transaction costs (commisions and bid-ask spread)

- Non-proportional costs (price-impact) affect profitability only for very large trades \$5 billion. This suggest that it would make sense for actual traders to engage in this strategy
- Value-weighted strategies are more profitable than equally-weighted after the price impact because of higher liquidity

- Recently, Frazzini, Israel, and Moskowitz (2014), using proprietary trading data, show that the capacity of momentum strategies is in the order of \$52 billion in the U.S. (\$89 billion globally)
- Incidentally, the same authors show that the capacity of Size and Value strategies is \$103 and \$83 billion in the US (\$156 and \$190 billion, globally)
- Instead, Novy-Marx and Velikov (2016) compute a capacity for an optimized version of Momentum of \$4.8 billion in the US, about 10 times smaller than above
- Momentum is the most puzzling anomaly given its significance and persistence
- The literature has evolved to use a four-factor model that includes the momentum factor plus the three Fama-French factors (Carhart, 1997, JF) in tests of new anomalies or managers' performance

Possible (behavioral) stories

1. Overreaction

   - 'Positive feedback traders' invest in winners and sell losers driving pricing away from fundamentals for some time
   - Long run reversal would be evidence in favor of this story

2. Underreaction

   - Investor do not fully incorporate information about short-term prospects of the firms, such as information in earnings announcement
   - Consistent with this story, Chan, Jegadeesh, and Lakonishok (1996, JF) find 'earnings momentum': one can predict future returns based on past earnings surprises. This is also consistent with the 'post-earnings-announcement drift' (Bernard and Thomas, 1989, J. of Accounting and Economics)

- The theoretical paper by Hong and Stein (1999, JF) keeps the two stories together by postulating slow information diffusion and two types of bounded rational traders:
  - News-watchers: cause underreaction because they don't extract other traders' information
  - Trend-chasers: cause overreaction because they condition their trades only on past price movements
- Hong, Lim, and Stein (2000, JF) find supporting evidence: momentum strategies are more profitable for firms with slower information diffusion (small firms and low analyst coverage)
- Behavioral stories need limits to arbitrage to survive. Moskowitz and Grinblatt (1999, JF) find that momentum is mainly an industry phenomenon (winner and loser industries). Hence, momentum returns contain industry factors which are hard to diversify away (no perfect arbitrage)

3. Daniel and Titman's Reply To F&F

The terms of the debate on the value premium

1. Rational Story (F&F):
   - Value companies load on a distress factor
   - The market requires high expected return because of high risk of this factor
   - Supporting evidence: high covariance of firms with similar B/M
2. Behavioral Story (LSV):
   - Investors extrapolate past performance into the future
   - Earnings mean-revert
   - Investors are suprised, which causes abnormal returns
   - It is possible that value firms covary, but risk premia are simply too high to be rationally justified by covariance with risk factor
   - Problem with last statement: measuring appropriate risk premium requires full specification of a consumption based model

A different approach

- Instead, Daniel and Titman (JF, 1997) take a different line of attack against the rational view
- They want to show that there is no such thing as a separate distress factor associated with B/M
- Moreover, they want to show that the value premium is purely due to the *characteristic* of being high B/M and not to covariance with any particular factor beyond the distress factor
- This approach gets around the problem of determining whether the risk premium is reasonable, which is inherent in LSV's argument

- Approach:
  - Identify firms with similar characteristics (B/M) but different loadings on risk factor (HML)
  - If covariance matters, they should have different average returns in spite of similar characteristic
  - They don't!
- Other Conclusion: high B/M covary not because of risk factor, but because they have similar properties (e.g. similar industry)

Three models for interpretation

1. The Null Hypothesis: closest to ICAPM and F&F's view

- There is a separate distress factor and the covariance structure changes as a function of performance:

$$
\begin{aligned}
R_{it} &= E\left(R_{it}\right) + \sum_{j=1}^{J} \beta_{i,j} f_{j,t} + \theta_{i,t-1} f_{D,t} + \varepsilon_{i,t} \\
\varepsilon_{i,t} &\sim \left(0, \sigma^2\right) \qquad f \sim (0,1)
\end{aligned}
$$

  where $\theta_{i,t-1}$ is the time-varying loading on the distress factor ($f_{D,t}$). You can approximate this loading with a firm's B/M and it changes over time as a function of a firm's performance: bad performance $\longrightarrow \theta_{i,t-1} \uparrow$

- High B/M firms move together because they all load on factor $f_{D,t}$

- The expected returns is:

$$
E_{t-1}\left(R_{i,t}\right) = R_{f,t} + \sum_{j=1}^{J} \beta_{i,j} \lambda_j + \theta_{i,t-1} \lambda_D
$$

So, if $\theta_{i,t-1} \uparrow \longrightarrow E_{t-1}\left(R_{i,t}\right) \uparrow$

2. Time-Varying Risk Premia: close to APT (still rational)

- The covariance structure is stable, but time-varying λ's. A factor's λ increases following a string of bad realizations. Typical sequence of events:
  - Oil factor has bad realizations
  - All firms in the oil industry become distressed (high B/M)
  - Individuals dislike oil industry because it is considered more risky ($\lambda_{oil} \uparrow$)
  - So, high B/M firms have high expected return
  - Also: high B/M firms move together because they all load on the same factors that brought them into distress
- There is no separate distress factor:

$$\begin{aligned} R_{it} &= E\left(R_{it}\right) + \sum_{j=1}^{J} \beta_{i,j} f_{j,t} + \varepsilon_{i,t} \\ \varepsilon_{i,t} &\sim \left(0, \sigma^2\right) \qquad f \sim (0,1) \end{aligned}$$

and the factor loadings are constant over time at firm level

- The expected returns is:

$$E_{t-1}\left(R_{i,t}\right) = R_{f,t} + \sum_{j=1}^{J} \beta_{i,j}\lambda_{j,t-1}$$

where the risk premia change over time: if $f_{j,t-1} \downarrow \longrightarrow \lambda_{j,t-1} \uparrow$

- There is an observable variable θ_i, which is used to form portfolios (e.g. B/M)
  - $f_{j,t-1} \downarrow \longrightarrow \theta_{i,t-1} \uparrow$ for the firms with high $\beta_{i,j}$
  - $\theta_{i,t-1}$ high occurs when λ_j is high and $E_{t-1}\left(R_{i,t}\right)$ is high

3. Characteristics based model: irrational story

- High B/M firms earn premia that are unrelated to covariance structure
- Incompatible with ICAPM and APT
- Covariances are stable over time, as in Model 2. But, unlike Model 2, risk premia do not

change over time

$$
\begin{aligned}
R_{it} &= E\left(R_{it}\right) + \sum_{j=1}^{J} \beta_{i,j} f_{j,t} + \varepsilon_{i,t} \\
E_{t-1}\left(R_{it}\right) &= a + b\theta_{i,t-1}\ , \qquad b > 0
\end{aligned}
$$

- Again, $\theta_{i,t-1} \uparrow$ if bad firm performance (value firms are distressed)
- But $\theta_{i,t-1}$ is not necessarily related to a factor performance
- There can be idiosyncratic reasons for a firm to become distressed or to do well
- So, there exists firms that load on distressed oil factor, but that are not in distress and their $E_{t-1}\left(R_{i,t}\right)$ is not high. And vice versa
- Implication: clever investor can form a well-diversified portfolio and earn risk premium without bearing factor risk (arbitrage opportunity)

Discriminate Model 1 Vs. 2 & 3

- If Model 1 is correct, the volatility of firms should increase when they become high B/M and decrease when they go back to normal:

$$\theta_{i,t-1} \uparrow \longrightarrow \; Var\left(R_{i,t}\right) \uparrow$$

  because they load more on the distress factor

- Instead, for the other two models the variance-covariance structure is stable

- Test: track volatility of stocks 5 years before and after they enter the high B/M portfolios

- Results in Table II: there is no significant change in volatility over time

- Possible objection: what if distress condition is permanent? (that is, $\theta_{i,t-1}$ is permanently high). You would not observe changing volatility

- Reply by D&T: indeed for these firms B/M and loading on HML changes substantially over time

- Conclusion: **there is no separate distress factor. Firms with similar B/M move together because they load on similar industry factors**

Discriminate Model 3 Vs. 1 & 2

- Model 3 postulates that characteristics (B/M) rather than risk factor loadings explain expected returns

- Methodology:
  - Construct 9 portfolios from a 3×3 sort by size and B/M
  - Each of these 9 portfolios is further split into 5 on the basis of the expected $\beta_{i,hml}$
  - Compute the expected $\beta_{i,hml}$ only on the basis of available information in June of year t: returns in months [-42;-7]
  - Under assumption of stable vcov matrix, the estimated $\beta_{i,hml}$ should predict post-formation $\beta_{i,hml}$
  - There is a concern that estimated betas are poor predictors of post-formation betas (measurement error)
  - They show that there is correspondence

- Test: look at α_i from three-factor model for these 45 portfolios
  - Models 1 & 2 predict: $\alpha_i = 0$
  - Model 3 predicts

$$\begin{aligned}
\alpha_i &= E(R_i) - \\
&\quad -\beta_{i,hml}\lambda_{hml} - \beta_{i,smb}\lambda_{smb} - \beta_{i,mkt}\lambda_{mkt} \\
&= \begin{cases} > 0 \text{ for low } \beta_{i,hml} \\ < 0 \text{ for high } \beta_{i,hml} \end{cases}
\end{aligned}$$

because $\lambda_{hml},\ \lambda_{smb}$, and λ_{mkt} are positive and no relation is expected between $E(R_i)$ and the factor loadings

Char Port		Factor Loading Portfolio					Factor Loading Portfolio				
BM	SZ	1	2	3	4	5	1	2	3	4	5
		α					$t(\alpha)$				
1	1	−0.58	0.14	0.06	−0.17	−0.67	−3.97	1.04	0.48	−1.34	−4.00
1	2	0.16	0.05	0.13	0.16	−0.08	0.94	0.47	1.12	1.33	−0.63
1	3	0.02	0.05	0.06	−0.06	0.28	0.15	0.42	0.50	−0.55	2.26
2	1	0.13	0.08	0.06	0.21	−0.31	1.05	0.87	0.73	2.13	−2.31
2	2	0.03	0.20	0.22	0.01	−0.31	0.24	1.71	2.05	0.14	−2.66
2	3	0.19	−0.08	0.05	−0.10	−0.07	1.13	−0.50	0.35	−0.67	−0.46
3	1	0.08	0.05	0.10	0.01	−0.47	0.70	0.55	1.06	0.10	−3.27
3	2	0.17	0.22	0.25	0.05	−0.31	1.25	1.67	1.94	0.36	−1.63
3	3	−0.01	0.16	−0.23	−0.12	−0.18	−0.04	1.13	−1.45	−0.74	−0.90
Average		0.02	0.10	0.08	0.00	−0.24	0.16	0.82	0.75	0.08	−1.51

- Results in Table V (1973-1993):
  - Consistent with Model 3, high $\beta_{i,hml}$ stocks have negative and significant α_i and vice versa
- Conclusion: characteristics (B/M) rather than covariance with risk factor ($\beta_{i,hml}$) explain expected returns
- Possible objection: B/M is a more refined proxy of loading on risk factor than $\hat{\beta}_{i,hml}$. In other words, controlling for B/M, the dispersion in estimated factor loadings is purely due to measurement error

Suggested Explanations

- LSV (1992) agency story: money managers are aware of superior returns of value stocks, but they don't hold them because growth stocks are simply easier to justify to their clients

- D&T: it is possible that investors incorrectly believed that value stocks were more risky because of their condition of distress. Data limitations in the 70's prevented them to realize that they are not more risky. Once more data become available, value premium should disappear

- Related story, but rational: Adrian and Franzoni (2009)
  - The beta of value stocks was indeed very high in the past (Franzoni, 2002)
  - If investors **learn about beta** from the data, and give a very high weight to the past in their filtering process, their current expectation of the true unobservable beta is high as well
  - The expected return on value stocks is high because it is a function of the expected beta

4. Idiosyncratic volatility is (inversely) priced, AHXZ, 2009, JFE

Motivation

- In previous paper, AHXZ (2006, JF) find that:
  - Low idiosyncratic volatility (ivol) stocks outperform high ivol stocks in the US by 1% per month
  - This spread in performance is not explained by F&F 3 factors, momentum, or sensitivity to aggregate volatility factor (e.g. VIX)
- In this paper they intend to show that:
  - Result holds out-of-sample (internationally). No data mining
  - Strong international comovement in the spread. Is it risk?
  - Rule out explanations based on a number of other asset pricing effects

Data and variable definitions

- For the US they use CRSP/Compustat. International data from Datastream
- In month t, ivol for stock i comes from time-series regression on daily data from month $t-1$

$$\begin{aligned} r_{i,t} &= \alpha_i + \beta_i MKT_t + s_i SMB_t + h_i HML_t + \varepsilon_{i,t} \\ t &= 1...T \end{aligned}$$

  ivol is the standard deviation of the fitted regression residuals $\hat{\varepsilon}_i$

- The estimated factor loadings are used as risk controls in the cross-sectional regressions
- They use local, regional, and global definitions of the F&F factors to get alternative measures of ivol

Fama and MacBeth regressions

- They run monthly cross-sectional regressions of monthly stock returns onto ivol, estimated factor loadings, and other controls

$$
\begin{aligned}
r_{i,t} &= c + \gamma ivol_{i,t} + \lambda'_{\beta}\beta_{i,t} + \lambda'_{z}z_{i,t} + u_{i,t} \\
i &= 1 \ldots N
\end{aligned}
$$

$\beta_{i,t}$ is a vector of estimated risk factor loadings
$z_{i,t}$ is a vector of stock characteristics including: size, BM, past-six-month return, country dummies

- Results (Table 2):

	Canada	France	Germany	Italy	Japan	U.K.	U.S.
Panel A: USD Denominated Returns							
Constant	1.723	0.602	0.753	0.425	0.948	0.480	1.746
	[3.68]	[1.13]	[1.87]	[0.76]	[1.25]	[1.03]	[3.83]
W-FF Idiosyncratic Volatility	-1.224	-1.439	-2.003	-1.572	-1.955	-0.871	-2.014
	[-2.46]	[-2.14]	[-3.85]	[-2.10]	[-5.18]	[-2.54]	[-6.67]
$\beta(MKT^W)$	0.344	0.059	0.277	-0.083	0.323	0.178	0.376
	[2.20]	[0.44]	[1.93]	[-0.32]	[3.12]	[1.46]	[4.52]
$\beta(SMB^W)$	0.009	0.015	-0.083	0.116	0.050	0.032	-0.049
	[0.12]	[0.17]	[-0.82]	[0.56]	[0.76]	[0.42]	[-1.19]
$\beta(HML^W)$	-0.070	-0.069	0.076	-0.221	-0.025	-0.077	-0.051
	[-0.95]	[-0.94]	[1.00]	[-1.98]	[-0.35]	[-1.30]	[-1.69]
Size	-0.253	-0.067	-0.044	-0.031	-0.132	-0.058	-0.157
	[-4.81]	[-1.08]	[-1.09]	[-0.47]	[-1.72]	[-1.16]	[-3.14]
Book-to-Market	0.369	0.569	0.176	0.239	0.550	0.365	0.282
	[3.68]	[4.59]	[1.35]	[1.48]	[3.84]	[4.46]	[3.87]
Lagged Return	0.014	0.001	0.003	0.001	-0.011	0.012	-0.001
	[3.57]	[0.10]	[1.01]	[0.15]	[-2.85]	[4.07]	[0.28]
Adjusted R^2	0.118	0.108	0.114	0.147	0.124	0.078	0.046
Percentiles of W-FF Idiosyncratic Volatility							
25th Percentile	20.8	21.4	16.3	21.5	23.1	13.9	25.0
75th Percentile	46.0	39.2	34.8	38.4	39.6	31.3	61.1
Economic Effect of Moving from the 25th to the 75th W-FF Idiosyncratic Volatility Percentiles							
25% $\rightarrow$ 75%	-0.31%	-0.26%	-0.37%	-0.27%	-0.32%	-0.15%	-0.73%

– Ivol is negative and significant for all countries

– Strong size and BM effects

– Magnitude: strongest in US, moving from 25th to 75th percentile of ivol causes a 0.73% drop in monthly returns

Portfolio results (time-series)

- Alternative methodology for two reasons
  - Robustness: portfolio aggregation reduces measurement error in risk controls
  - Look for international comovement and possibly risk factor
- Sort stocks by ivol at regional level
  - Each month, within each country, form 5 value-weighted porfolios based on quintiles of ivol
  - Aggregate portfolios at the regional level
  - Form "5−1" strategies: long highest ivol portfolio and short lowest ivol portfolio
  - Look at alphas from 3 factor model and from model that includes US 5−1 portfolio (VOL^{US})
- Results (Table 7):

	Alpha	MKT^W	SMB^W	HML^W	VOL^{US}	Adjusted R^2
Panel A: Using the W-FF Model						
U.S. (VOL^{US})	-1.952	0.733	1.307	-0.311		0.51
	[-5.59]	[8.56]	[13.1]	[-1.88]		
Europe	-0.723	0.456	0.433	0.004		0.29
	[-3.01]	[7.72]	[6.32]	[0.04]		
Far East	-0.529	0.339	0.699	-0.087		0.28
	[-1.84]	[4.82]	[8.54]	[-0.64]		
G7	-1.353	0.622	1.028	-0.220		0.57
	[-5.46]	[10.2]	[14.6]	[-1.88]		
G7 Excluding U.S.	-0.651	0.432	0.618	-0.087		0.37
	[-2.77]	[7.49]	[9.23]	[-0.79]		
All	-1.307	0.596	0.966	-0.189		0.58
	[-5.69]	[10.6]	[14.8]	[-1.75]		
All Excluding U.S.	-0.670	0.428	0.597	-0.050		0.41
	[-3.16]	[8.24]	[9.89]	[-0.50]		
Panel B: Using Only VOL^{US}						
Europe	0.134				0.370	0.42
	[0.63]				[14.1]	
Far East	0.130				0.271	0.16
	[0.43]				[7.29]	
G7	0.121				0.723	0.90
	[1.04]				[50.6]	
G7 Excluding U.S.	0.176				0.362	0.37
	[0.77]				[12.8]	
All	0.081				0.673	0.89
	[0.71]				[47.6]	
All Excluding U.S.	0.148				0.348	0.40
	[0.71]				[13.6]	

– Panel A: negative and significant alphas for all regions, consistent with cross-sectional result

– Strongest result in US: -1.95% monthly alpha

– Panel B: the VOL^{US} portfolio comoves significantly with the returns of the ivol portfolios in other regions (look at t-stats and R-squared)

– The VOL^{US} portfolio makes the alphas insignificant: a risk factor? Maybe. But they don't push is too hard because they don't have an economic motivation for it

An explanation of the IVOL anomaly

- Stambaugh, Yu, and Yuan (JF, 2015) provide an explanation for the persistence of this anomaly based on:
  - Arbitrage risk
  - Arbitrage asymmetry
- *Arbitrage risk* is the fact that it is more difficult to arbitrage high-volatility stocks
- *Arbitrage asymmetry* is the notion that there is much more capital in the market that is willing to go long in underpriced stocks than short in overpriced stocks
- For example, mutual funds typically are long-only investors. Hedge funds' hold a much smaller size of the market
- According to the authors, these two facts lead to:
  - Persistence in overpricing for high-IVOL stocks due to arbitrage risk

- Persistence in underpricing for high-IVOL stocks due to arbitrage risk
- In other words, mispricing, whether positive or negative, should be stronger in high-IVOL stocks because they are more difficult to arbitrage
- But, due to arbitrage asymmetry, the overpricing of high-IVOL stocks is larger than the underpricing of high-IVOL stocks
- Hence, on average, high-IVOL stocks tend to be overpriced stocks

• Note that, in this explanation, mispricing is exogenous and due to noise traders. IVOL is a *limit to arbitrage*

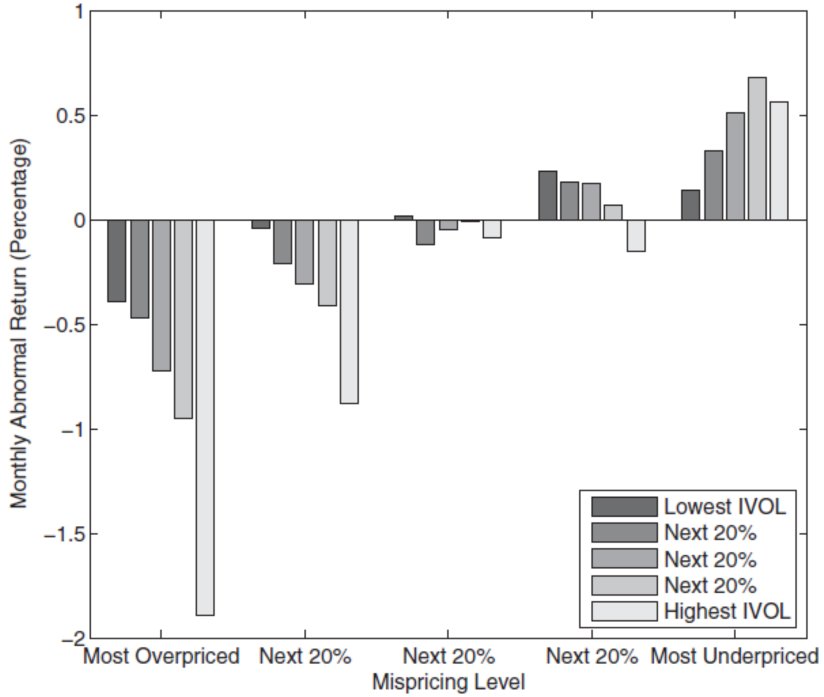
Supporting evidence

- The authors combine 11 anomalies to define a composite mispricing measure
- See the paper for the list of anomalies that include: momentum, profitability, investment, accruals, etc.
- Then, they rank stocks according to each anomaly (higher rank for more overpricing) and take the arithmetic average of the ranks
- They double sort stocks by IVOL and the mispricing metric and form portfolios
- Here are the portfolio average abnormal returns:

	Highest IVOL	Next 20%	Next 20%	Next 20%	Lowest IVOL	Highest –Lowest	All Stocks
Most Overpriced	−1.89	−0.95	−0.72	−0.47	−0.39	−1.50	−0.81
(Top 20%)	(−12.05)	(−7.39)	(−4.90)	(−3.62)	(−3.04)	(−7.36)	(−8.14)
Next 20%	−0.88	−0.41	−0.31	−0.21	−0.04	−0.84	−0.23
	(−5.86)	(−3.36)	(−3.00)	(−2.08)	(−0.44)	(−4.41)	(−3.88)
Next 20%	−0.09	−0.01	−0.05	−0.12	0.02	−0.10	−0.07
	(−0.53)	(−0.09)	(−0.48)	(−1.29)	(0.18)	(−0.53)	(−1.47)
Next 20%	−0.15	0.07	0.17	0.18	0.23	−0.38	0.18
	(−0.80)	(0.63)	(1.87)	(2.33)	(3.22)	(−1.78)	(4.45)
Most Underpriced	0.56	0.68	0.51	0.33	0.14	0.41	0.28
(Bottom 20%)	(3.27)	(4.91)	(5.02)	(4.10)	(2.04)	(2.16)	(5.67)
Most Overpriced –	−.44	−1.63	−1.23	−0.81	−0.53	−1.91	−1.09
Most Underpriced	(−11.07)	(−8.65)	(−6.43)	(−5.02)	(−3.43)	(−7.62)	(−8.05)
All Stocks	−0.69	−0.12	−0.00	0.05	0.08	−0.78	
	(−6.09)	(−1.56)	(−0.01)	(1.07)	(1.86)	(−5.50)	

- Consistent with arbitrage risk, there is more overpricing for high-IVOL stocks (red box)
- Similarly, there is more underpricing for high-IVOL stocks (blue box)
- However, consistent with arbitrage asymmetry, the overpricing is larger in absolute value than the underpricing (i.e. -1.50 vs. 0.41)
- Hence, when you aggregate across all stocks, the overpricing of high-IVOL stocks prevails and you find a decreasing pattern between returns and IVOL (green box)

- You can also see the result graphically



5. Betting against beta, Frazzini and Pedersen, 2014

Motivation

- Beta does not explain the cross-section of average returns
- CAPM assumes that investors can freely borrow
- However, some investors cannot borrow and others have limited leverage (e.g. mutual funds)
- Can this fact explain the failure of beta to explain average returns?
- Note: Black (1972) had a version of CAPM with borrowing constraints
- The contribution of this paper:
  - Argue through a model with constrained agents that low beta stocks earn a premium
  - Develop a factor which is long low beta stocks and short high beta stocks
  - Show that the factor earns positive risk adjusted returns
  - Across asset classes

Model intuition

- Some agents would like to leverage up the market, but they cannot
- So, they get the high expected returns that they want by investing in high beta stocks
- High beta stocks' prices are bid up and expected returns decrease
- Low beta stocks pay higher expected returns
- Agents that can use leverage, buy low beta stocks and short high beta stocks
- These agents face margin requirements, so they cannot borrow as much as they would like
- So they cannot fully reestablish the security market line
- The security market line is flatter: higher intercept than the r_f and lower slope than $E(r_m) - r_f$
- As a consequence, low beta stocks earn higher risk adjusted returns than high beta stocks
- NOTE: constrained agents have to be very influential in price formation to affect expected returns

The betting-against-beta (BAB) factor

- It combines a long position in low beta stocks and short position in high beta stocks
- Zero beta
- Zero net investment
- For US equity:
  - pre-ranking betas on daily data at the stock level with one year of data (Dimson 1979 adjustment for non-synchronous trading)
  - Every month, form two portfolios based on pre-ranking beta (high and low beta stocks): r^H_{t+1}, r^L_{t+1}
  - BAB:

$$r^{BAB}_{t+1} = \frac{1}{\beta^L_t}\left(r^L_{t+1} - r_f\right) - \frac{1}{\beta^H_t}\left(r^H_{t+1} - r_f\right) \tag{1}$$

  where β^L_t and β^H_t are weighted averages of security betas

- β_t^L =0.7 and β_t^H =1.5 for US equity
- The portfolio has zero beta in expectation

Empirical evidence

	P1 (Low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)	BAB Factor
Excess return	**0.99**	**0.90**	**0.92**	**0.98**	**1.04**	**1.12**	**1.07**	**1.07**	**1.03**	**1.02**	**0.71**
	(5.90)	(5.24)	(4.88)	(4.76)	(4.56)	(4.52)	(4.08)	(3.71)	(3.32)	(2.77)	(6.76)
CAPM alpha	**0.54**	**0.39**	**0.35**	**0.35**	**0.34**	**0.37**	**0.26**	0.19	0.09	-0.05	**0.69**
	(5.22)	(4.70)	(4.23)	(4.00)	(3.55)	(3.41)	(2.45)	(1.54)	(0.65)	-(0.29)	(6.55)
3-factor alpha	**0.38**	**0.25**	**0.19**	**0.18**	**0.15**	**0.14**	0.04	-0.07	**-0.18**	**-0.36**	**0.66**
	(5.24)	(4.43)	(3.69)	(3.62)	(2.65)	(2.49)	(0.75)	-(1.06)	-(2.45)	-(3.10)	(6.28)
4-factor alpha	**0.42**	**0.32**	**0.24**	**0.24**	**0.24**	**0.25**	**0.17**	**0.12**	0.04	-0.07	**0.55**
	(5.66)	(5.67)	(4.55)	(4.63)	(4.20)	(4.58)	(3.00)	(1.98)	(0.61)	-(0.59)	(5.12)
5-factor alpha*	**0.23**	**0.23**	**0.17**	**0.16**	**0.16**	**0.20**	**0.22**	0.06	0.11	0.01	**0.46**
	(2.37)	(3.00)	(2.28)	(2.13)	(2.08)	(2.76)	(2.86)	(0.69)	(1.08)	(0.07)	(2.93)
Beta (ex ante)	0.57	0.75	0.84	0.92	0.99	1.06	1.14	1.23	1.36	1.64	0.00
Beta (realized)	0.75	0.86	0.97	1.07	1.18	1.28	1.37	1.50	1.60	1.82	0.03
Volatility	18.2	18.7	20.6	22.4	24.7	27.0	28.4	31.5	33.8	40.0	11.5
Sharpe ratio	0.65	0.58	0.54	0.52	0.50	0.50	0.45	0.41	0.37	0.31	0.75

- US Equity: ten beta portfolios + BAB factor
- The average excess returns of the ten beta portfolios are similar
- Hence, if you leverage up the low-beta portfolio and leverage down the high-beta portfolio, you obtain a positive BAB spread, with about zero exposure on the market
- The BAB factor has significant alpha relative:

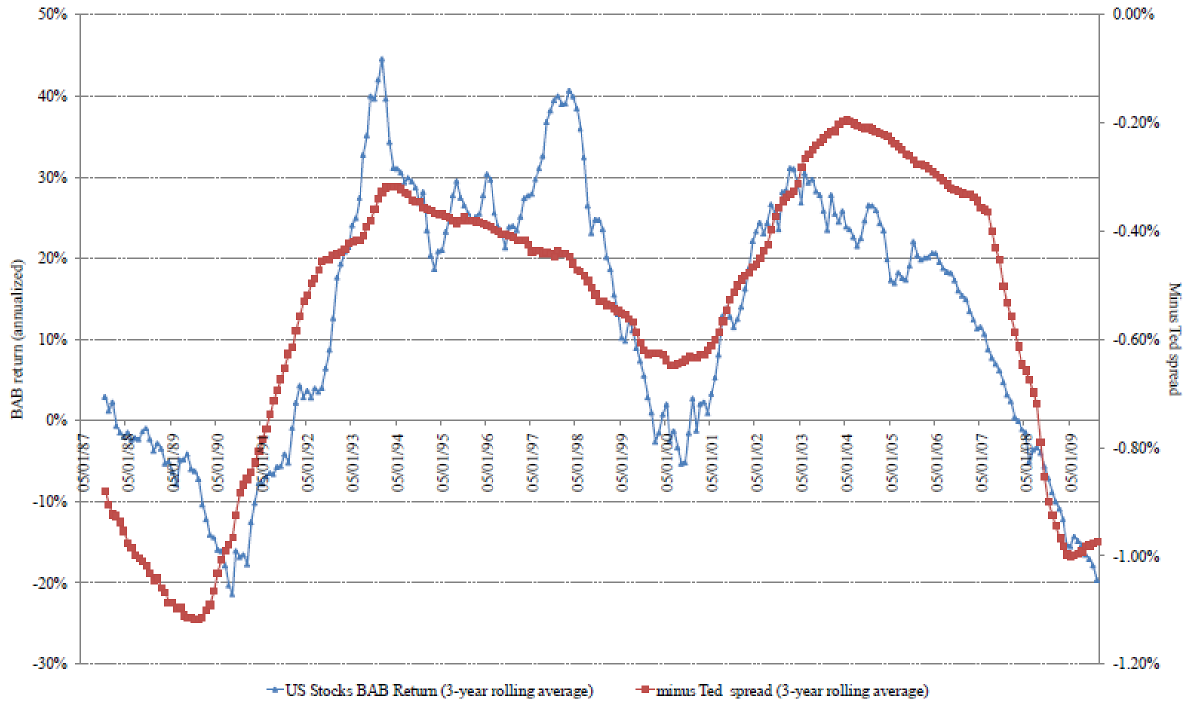
  - Market factor
  - Fama and French
  - FF + Momentum (Carhart, 1997)
  - FF + Momentum + Liquidity (Pastor and Stambaugh, 1993)

- Low volatility: 11.5% annually

- Annualized Sharpe Ratio: 0.75 (comparable to momentum)

Relation to funding liquidity

- The TED spread (3 month LIBOR - 3 month TBill) measures funding liquidity
- That is, the availability of trading capital to speculators (see Brunnermeier and Pedersen, 2009, Garleanu and Pedersen, 2011)
- When funding liquidity tightens, leveraged investors sell low-beta stocks
- So, the BAB factor, which is long low-beta stocks, is expected to have negative realizations
- Confirmed in the data



50%
40%
30%
20%
10%
0%
-10%
-20%
-30%
BAB return (annualized)
0.00%
-0.20%
-0.40%
-0.60%
-0.80%
-1.00%
-1.20%
Minus Ted spread
05/01/87
05/01/88
05/01/89
05/01/90
05/01/91
05/01/92
05/01/93
05/01/94
05/01/95
05/01/96
05/01/97
05/01/98
05/01/99
05/01/00
05/01/01
05/01/02
05/01/03
05/01/04
05/01/05
05/01/06
05/01/07
05/01/08
05/01/09
US Stocks BAB Return (3-year rolling average)
minus Ted spread (3-year rolling average)

Portfolio predictions

- Mutual funds and individual investors typically do not use leverage $\longrightarrow$ They should buy high beta stocks
- LBO firms and Berkshire Hathaway (Warren Buffets' firm) can use leverage $\longrightarrow$ They should buy low beta stocks
- Confirmed in the data

Critical assessment

- Given that the SML does not work, the BAB factor in equation (1) is built to have a positive alpha:
  - All portfolios have the same $E(r_i)$
  - Then, if you divide by $\beta_L < 1$ you get something large from which you subtract something small (because it is divided by $\beta_H > 1$)
  - This is simply Black's (1972) idea to exploit the fact that the SML is too flat
  - In this sense, the idea is not original
- "Betting Against Betting Against Beta", Novy-Marx and Velikov, 2018
  - The rank-weighted portfolios overweight nano-caps (those in the first decile of the size distribution)
  - Value-weighted monthly performance drops from 1% to 0.56%
  - The problem with overweighting small stocks is transaction costs: 60 bps/month (computed using Novy-Marx and Velikov, 2016, RFS)

- Net-of-transaction-costs monthly performance is 0.48%/month
- The strategy loads significantly on the investment and profitability factors (see below)
- The net-of-transaction-costs alpha of BAB from five-factor model is 0.06%/month

Fama and French: the redux

- Fama and French in a series of papers, most notably JFE (2015) and RFS (2016), increase the set of factors in their asset pricing model

- They include two additional factors:
  - RMW: long in high-profitability (robust) and short in low-profitability (weak) firms
  - CMA: long in low-investment (conservative) and short in high-investment (aggressive) firms

$$\begin{aligned} R_{i,t} &= a_i + b_i \left(R_t^{mkt} - R_t^f \right) + h_i HML_t \\ &\quad + s_i SMB_t + r_i RMW_t + c_i CMA_t + e_{i,t} \end{aligned}$$

- The goal is to account for the anomalies related to:
  - Profitability: more profitable firms have higher average returns (Novy-Marx, 2013, JFE)
  - Investment: high investment firms have lower average returns (Titman, Wei, Xie, 2004, JFQA)

- The inspiration comes from the dividend discount model, which is an identity, and can be written to define the internal rate of return (IRR) r

$$M_t = \sum_{\tau=1}^{\infty} \frac{E\left(d_{t+\tau}\right)}{(1+r)^{\tau}} \tag{2}$$

where r is the 'average' expected return over the life of the stock. More properly, r is the IRR of the stock, M_t is the share price, d_t is the per-share dividend

- Equation (2) can be re-written by expressing dividends as the difference between earnings Y and the change in book value dB (i.e. retained earnings, which proxy for investment)

$$M_t = \sum_{\tau=1}^{\infty} \frac{E\left(Y_{t+\tau} - dB_{t+\tau}\right)}{(1+r)^{\tau}}$$

- Finally, you can divide by book value of equity to obtain

$$\frac{M_t}{B_t} = \sum_{\tau=1}^{\infty} \frac{E\left(Y_{t+\tau}/B_t - dB_{t+\tau}/B_t\right)}{(1+r)^{\tau}} \tag{3}$$

- From Equation (3), you can see that:

  - Keeping everything else constant, a higher BM ratio implies a higher expected return
  - Keeping everything else constant, higher profitability implies a higher expected return
  - Keeping everything else constant, higher investment implies a lower expected return

- Importantly: this equation is just an identity and has nothing to say about the risk-based or behavioral determinants of expected returns

- However, this equation can serve as an explanation for the findings that BM, profitability, and investment are related to average returns. **If the tests fail to adjust for the true sources of risk**, then you should expect this relation to show up

- Fama and French take inspiration from this argument to construct a five factor model that accounts for profitability and investment

- In these papers, the authors do not make any claim that these variables are proxies for hidden risk factors. They simply argue that these factors 'work well' in explaining returns

- Hence, the quest for a risk-based explanation of anomalies seems to have become of secondary importance even for Fama and French

- They show that their model has explanatory power for other anomalies, beyond profitability and investment
  - Low β firms (e.g. Frazzini and Pedersen, 2014) have positive exposure to RMW and CMA
  - High-share-repurchase firms (e.g. Loughran and Ritter, 1995) have positive exposure to RMW and CMA
  - Low-volatility firms (e.g. Ang et al., 2006) have positive exposure to RMW and CMA
- Instead, the five-factor model does not manage to explain the Accruals anomaly (Sloan, 1996) and the Momentum anomaly (Jegadeesh and Titman, 1993)
- Also interesting, once RMW and CMA are introduced, the additional explanatory power of HML goes to zero
- Overall, the authors argue, the five-factor model reduces the number of independent anomalies that deserve an explanation
- To conclude, the impression is that even Fama and French have fallen back on the option of certifying the outstanding anomalies. Providing a risk-based, economically founded, explanation does not seem to be the goal at this point

- Hou, Xue, Zhang (2015, RFS) actually claim to be the first to have introduced investment and profitability in a factor model

- Including the market and size factors, they end up with a four-factor model, which they label the q-model, from the Q-theory of investment:

$$\begin{aligned} E\left[r^i\right] - r^f &= \beta^i_{MKT} E\left[MKT\right] + \beta^i_{ME} E\left[r_{ME}\right] \\ &\quad + \beta^i_{I/A} E\left[r_{I/A}\right] + \beta^i_{ROE} E\left[r_{ROE}\right] \end{aligned}$$

- Their justfication for including investment and profitability is the same as Fama and French's, although they derive it within a neoclassical model in which firms choose investment optimally, subject to convex adjustment costs (Q-theory, by Tobin)

- They consider 80 different anomalies

- They find that q-model does a better job at pricing these anomalies than the Fama and French (1993) and Carhart (1997) models

A Neoclassical Model for Investment

- From the first order condition for investment, they derive the following intuitive equation

$$E_t\left[r^S_{t+1}\right] = \frac{E_t\left[\Pi_{t+1}\right]}{1 + a\left(I_t/A_t\right)} \tag{4}$$

  where Π_{t+1} is the firm profitability per dollar invested, $1 + a\left(I_t/A_t\right)$ is the cost of investing, i.e. \$1 for the investment good + \$$a\left(I_t/A_t\right)$ for the marginal adjustment cost, and r^S_{t+1} is the stock return

- The equation tells you that investment occurs to the point where discounted profits from one unit of additional investment equate the cost of investment. The discount rate is the cost of capital, or expected stock return

- From this equation, you see that
  - Keeping profitability constant, firms with higher investment have lower expected returns
  - Keeping investment constant, firms with higher profitability have higher expected returns

- Simply, if you invest a lot, it must be that your cost of capital is lower (lower expected return on equity)

- Also, if you are profitable and do not invest a lot, it must be that your cost of capital is high

Relation to other anomalies

- The investment channel helps explain other anomalies that are related to investment
  - *Net stock issues, composite issuance.* Firms that raise capital, then put this capital to work via investments.
  - *Boot-to-market and other valuation ratios.* Firms with large growth options investment (i.e. with high valuation ratios) tend to invest more and have lower expected returns
- The profitability channel helps explain other anomalies that are related to firm profitability
  - *Momentum, Earnings Momentum.* Firms with high profits tend to have had positive returns and positive earnings shocks, they have high expected returns
  - *Distress related anomalies.* Firms in distress have low profits and low expected returns

A Caveat

- Note that equation (4) is derived from the q-theory in partial equilibrium
- It does not contain any prediction on whether stocks with similar profitability or investment move together
- Remember that comovement is a necessary condition for a factor to be priced. If they did not move together, the risks could be diversified
- Moreover, you need these aggregate risks to matter in terms of driving the consumption of the marginal investor for them to appear in the SDF
- In sum, the model boils down to a convenient reduced-form equation, but it does not go all the way to deriving the asset pricing foundations of the new factors

Portfolio Properties

- 2 size, 3 Investment/Assets, 3 ROE groups. In total, 18 portfolios. ROE portfolios rebalanced monthly, size and I/A rebalanced annually
- Sample period January 1972-December 2012
- The size factor has average monthly return of 0.31% (t=2.12). Better than Fama and French's SMB (0.19, t=1.35). The difference is that in this paper the size factor is the difference between 9 portfolios on the long and 9 on the short side. It has 95% correlation with SMB
- The investment factor has average monthly return of 0.45% (t=4.95). FF alpha is 0.33 (t=4.85). Correlation with HML of 0.69. Investment captures value effects
- The ROE factor has average monthly return of 0.58% (t=4.81). With FF alpha of 0.77% (t=6.94). Correlation with UMD (momentum) of 0.50
- The large alphas of the new factors imply that even the stricter requirements on statistical significance advocated by Harvey, Liu, and Zhu (2013), i.e. t-stat>3, are satisfied

- Importantly, the alphas of HML and UMD in the q-factor model are 0.06% and 0.13% and insignificant. Therefore, the q-factor model subsumes the Fama and French (1993) and Charart (1997) factors but not vice versa

The Playing Field

- They run time series regressions of 38 anomalies (top decile - bottom decile) on a four factor model
- Size factor is included to make model comparable to other models, but size does not help in improving pricing performance
- Market factor is included to account for time series variation in returns, given that other factors are long-short and to a large extent market neutral
- Importantly, in constructing anomaly portfolio drawing from previous literature, they use NYSE breakpoints and value-weighting
- These two choices reduces the extent of significance of the 80 anomalies that they consider. Only 38 remain significant
- Note: Fama and French point out that the smallest quintile of stocks based on NYSE breakpoints (the so-called microcaps) contains 60% of stocks, but only 3% of market capitalization
- Conclusion: the extent of previous anomalies was exaggerated and concentrated in small illiquid stocks

Main Results

- They evaluate peformance based on size and significance of the anomaly's alpha (top minus bottom portfolio), the average absolute alpha across the ten anomaly portfolios, and the GRS statistic

- Except for the Accrual anomaly, the q-factor model in general outperforms the Fama and French and Carhart models

- Importantly, *it does a better job in the momentum category* as well as in the investment and in the profitability categories

- The three models have similar performance in the value vs. growth category

- The Total Volatility anomaly (IVOL does not survive value-weighting and NYSE breakpoints) is also partly explained, both the I/A and ROE factors are significant explanatory variables

Conclusion

- The maximum Sharpe Ratio that can be achieved by combininig the q-factors using the formula

$$\left(\mu_f' V_f^{-1} \mu_f\right)^{1/2}$$

is 0.43, which is high, but not extremely high to the point that no risk would be involved

- Hence, this Sharpe Ratio denotes that the volatility of these factors is not completely diversified away, suggesting that there is comovement in the stocks in the portfolios

- Comovement is a necessary condition for a risk-based explanation of the factors

- However, the authors do not want to push hard on a risk based story. They are agnostic about it

- Indeed, they argue that comovement could arise also if there is systematic variation in sentiment (i.e. irrational behavior) that is correlated with investment and profitability

- E.g. investors could all become too optimistic about companies that invest a lot. This would predict lower returns going forward, but these stocks' returns would move together

- In a follow-up paper, Hou, Mo, Xue, Zhang (2019 RF) develop a q^5-factor model, including expected growth factor, which is the expected one-year-ahead change in investment to assets

- They show that this model subsumes the Fama and French five and six (2015, 2018) factor models and the Stambaugh and Yuan (2017) four-factor model

- Fama and French (2018, JFE): Fama and French five factors + momentum

- Stambaugh and Yuan (2017, RFS): Market, Size, 2 Mispricing factors resulting from the aggregation of anomalies around clusters of comovement

Replication Crisis

- Hou, Xue, and Zhang (2019, RFS) are inspired by the new trend towards replicating past empirical results that follows the 'Replication Crisis' in Psychology
- They expand the set of anomalies that they consider to 452
- Most anomalies fail to hold up to currently acceptable standards for empirical finance
- With microcaps mitigated via NYSE breakpoints and value-weighted returns, 65% of the 452 anomalies cannot clear the single test hurdle of the absolute t-stat of 1.96
- Imposing the higher hurdle of 2.78 for multiple tests (as advocated by Harvey, Liu, and Zhu 2013) raises the failure rate to 82%
- Even for replicated anomalies, their economic magnitudes are much smaller than originally reported
- Overall, the study points to the important effects of publication bias (only significant results get published and there's no incentive to do replication) and *p-hacking* (authors have strong incentives to search many specifications and variable definitions to eventually find significant results. This behavior should raise the threshold for statistical significance)

McLean and Pontiff (JF, 2016)

- They look at the persistence of anomaly returns out-of-sample (i.e., after the end of the paper's sample, but before publication) and after publication
- They look at 97 different predictors from 79 different academic studies
- They find that out-of-sample returns from predictors decline by on average 26%
- This decline is to be imputed to statistical bias: the fact that investors do in-sample search for the predictor with the best performance
- They also find that post-publication returns from predictors decline by an additional 32%
- This decline is to be imputed to the fact that arbitrageurs learn about the profitability of these trading strategies and take advantage of them
- Overall, predictability declines after publication by 58% (26%+32%)

- The conclusion is that most of the predictors capture true mispricing, which is arbitraged away over time

- Also remember the learning explanation (Martin and Nagel, 2019) that we discussed in connection with market efficiency

- If the predictability was based on risk, it would survive out-of-sample

- Persistence in predictability is stronger for stocks with higher limits of arbitrage: systematic noise trader risk (DSSW, 1990, see later) and idiosyncratic volatility (Pontiff, 1996, 2006)

- Short interest in low-return anomalies increases after publication: evidence of arbitrage activity

- Before publication, stocks in the same anomaly comove, consistent with theories that postulate irrational comovement for stocks in the same style ('style investing', Barberis and Shleifer, 2003)

- After publication, correlation across different anomaly returns increases, consistent with these stocks becoming part of arbitrageurs' portfolios